

A Quarterly Econometric Model of the United States:

A Progress Report

IN RECENT years economists have made increased use of a relatively new tool for analyzing the behavior of the overall economy—the econometric model. This kind of model—of which there are now a considerable number—attempts to depict in a set of equations the essential quantitative relationships that determine the behavior of such magnitudes as output, income, employment, and prices. Econometric models have been used for forecasting, estimating the quantitative impact of alternative Government policies, and testing various hypotheses about the nature of the business cycle.

This article presents a quarterly model of the U.S. economy that has been developed by the Office of Business Economics. It is a variant of one constructed under the direction of Professor Lawrence R. Klein at the Wharton School of Finance and Commerce of the University of Pennsylvania. The original model, consisting of 34 equations, was designed primarily as a forecasting instrument.¹ In the model's further development at OBE, this characteristic has been maintained.

It should be made quite clear that this article is a progress report on work that must be regarded as experimental. Forecasting business activity is hazardous whatever technique is used and the econometric technique is no exception. This article is published with the intention of fostering the

progress of research in this field; no predictions of the future will be presented.

The first part of this article deals with the nature of econometric models.

The second describes the OBE model. The third reports the results of tests that show how well the model has depicted the behavior of the U.S. economy since the Korean war.

Econometric Models

The characteristics of an econometric model and the steps involved in its construction and use will be explained by reference to a simplified version of actual models. The following set of six equations constitutes a complete model, although hardly a realistic one, and will serve to illustrate the main points.

- (1) $C_t = \alpha_0 + \alpha_1 Y_t + \alpha_2 C_{t-1} + u_{1t}$
- (2) $I_t = \beta_0 + \beta_1 P_t + \beta_2 K_{t-1} + u_{2t}$
- (3) $W_t = \gamma_0 + \gamma_1 Y_t + \gamma_2 t + u_{3t}$
- (4) $Y_t = C_t + I_t + G_t$
- (5) $P_t = Y_t - W_t$
- (6) $K_t = K_{t-1} + I_t$

The variables included in the above equations are defined as:

C = Consumption
 Y = Income (net product)
 W = Wage income
 P = Nonwage income
 I = Net investment
 K = Net capital stock at end of period
 t = time
 G = Government expenditures on goods and services

u_1, u_2, u_3 = disturbance terms

The subscript t refers to a given time period; $t-1$ to the previous period.

The first equation states that consumption in the current period depends on the same period's income and on consumption in the previous period. Net investment, represented in equation (2), is determined by nonwage income earned in the current period and by the net capital stock available at the end of the previous period. Wages, in equation (3), are related to income and time. The latter stands for factors that are not further specified and that affect the economic variables gradually and persistently. The remaining three equations, called identities, are definitional statements and are needed to complete the model. Total income (or net product) is defined in equation (4) as the sum of consumption, net investment, and government expenditures. (The items that in the real world constitute differences between net income and product are omitted.) Nonwage income is the difference between total income and wage income (equation 5), and the net capital stock at the end of the current period is equal to the last period's stock plus current net investment (equation 6).

The first three equations contain, besides the explanatory variables on the

1. See Lawrence R. Klein, "A Postwar Quarterly Model: Description and Applications," *Models of Income Determination* (Princeton University Press, 1963), pp. 11-57. See also Lawrence R. Klein and Joel Popkin, "An Econometric Analysis of the Postwar Relationship Between Inventory Fluctuations and Changes in Aggregate Economic Activity," in Joint Economic Committee, *Inventory Fluctuations and Economic Stabilization, Part III*, 87th Congress, 1st Session, 1961, pp. 71-89 (U.S. Government Printing Office, 1961.)

right-hand side, the variables u_1 , u_2 , and u_3 respectively. These terms, called disturbance terms, are included in explicit recognition of the fact that the other variables cannot fully explain movements of the dependent variables on the left-hand side. Assuming that no significant variables have been omitted, the disturbance terms can be regarded as reflecting random elements representing the net effect of a host of unknown and unpredictable factors. Ideally they are small so that the remaining ("systematic") part of each equation accounts for most of the movements in the dependent variable. The last three equations, because they hold by definition, contain no disturbance terms.

The following section explains how the equations of a model are constructed. A later section shows how they are solved and how a model is used.

Constructing the model

As a basis for an econometric model the investigator must, first of all, establish a conceptual framework that sets forth the way in which he believes the economy to work. In the example, for instance, there are three components of final demand—consumption, investment, and government expenditures—that are determined by different sets of factors. Total demand, made up of the three components, calls forth production of an equal amount; this implies that there are no resource limitations. On the income side, it is assumed that wages are systematically explained while nonwage income is residually determined.

Such a framework does not, of course, fix the exact character of the model. There is wide latitude left with respect to the particular form a model may take. For instance, it may be highly aggregative, containing only a few variables and equations, like the illustrative example, or it may be very disaggregative, containing many.² The

choice depends in part on how much the model builder wishes to explain and upon how much detail he thinks is needed to make a model perform reasonably well. Models also vary with respect to the length of the unit time period; in practice, this period has varied from a quarter to a year.

There is also considerable latitude at the next step of model building—the formulation of the component equations. In the example, the first three equations represent the kind over which the model builder has discretion, for they embody hypotheses regarding economic behavior; the identities arise naturally as logical requirements for completeness.

The investigator selects equations as a result of testing various economic hypotheses on empirical data. More specifically, he uses regression methods in determining how well the hypotheses fit the data for some selected time period. In the process, he obtains estimates of the parameters, that is, values of the α 's, β 's, and γ 's. Equations embodying given hypotheses may be entertained during the fitting and testing stage only to be subsequently discarded because they explain the historical data poorly. Others may be discarded even if they fit such data well, because they do not provide adequate predictability when tested beyond the period of fitting.

The testing of hypotheses with actual economic magnitudes and the selection of a workable set of equations are the most important tasks of the model builder. He must decide not only which variables are to be included in each equation but also what form the variables are to take. Together, these two decisions constitute what is called *specification*. For instance, in the example, the consumption equation might have contained, instead of total income, W and P as separate variables. In specifying equations, the model builder is normally guided by economic theory, institutional knowledge of the economy, and results obtained by other research workers. But there remains a wide area of freedom for exercising ingenuity, which is reflected in different specifications among different models for equations explaining the same dependent

variable. The task of specification is never really finished since new research may suggest other relevant variables and new forms. Revised specification may also be called for because of basic changes in the economy that make the old equations inapplicable.

Using the model

After the equations have been decided upon and the parameters estimated, the model can be tested as a whole and applied. This means solving the set of equations for values of the unknown or *endogenous* variables. First, values of the inputs to the model are obtained. These inputs are all those variables assumed to be known at the time the model is to be processed; in the case of the illustrative model, these are the prior period's consumption and capital stock, time, and government expenditures. These variables are referred to as *predetermined*, and they include both lagged values of endogenous variables and other magnitudes, such as time and government expenditures, designated as *exogenous*. Variables are regarded as exogenous if they are believed to be determined essentially outside the economic system. However, certain other variables may be treated as exogenous if they cannot be adequately predicted by regression equations or if making them endogenous would require a substantially enlarged model.

After the predetermined values have been introduced into the equations, the entire set is solved simultaneously, and the outputs—the endogenous variables—are obtained. In the example, there are six independent equations and six unknowns, the current endogenous variables C_t , Y_t , I_t , W_t , P_t , and K_t . Thus, the model is complete and can be solved. The disturbance terms are also unknowns, but are assumed to be zero in accordance with their statistically expected value. Clearly, the values determined for each unknown depend on both the magnitude of the inputs and the coefficients (the estimates of the α 's, β 's, and γ 's).

When the model is used for forecasting purposes, it is apparent that in addition to the lagged values, projections of all the exogenous variables

2. Recently developed models vary in size from a five-equation model (see I. Fritland and P. Teubman, "A Semi-Annual Forecasting Model," *Review of Economics and Statistics*, August 1964, pp. 229-236) to the very large Brookings-SSRC model, which has over 300 equations in the complete version. See J. Duesenberry, G. Fromm, L. Klein, and E. Kuh (eds.), *The Brookings Quarterly Econometric Model of the United States* (Rand McNally and Company, 1960).

must be included as inputs. In the illustrative model, there are only two such variables, time and government expenditures. Only the latter, of course, is not known with certainty. With all predetermined values introduced, a solution is obtained for the first of the future time periods. Forecasts beyond the first period are made by further projections of exogenous variables and the use of needed outputs of earlier solutions as lagged endogenous variables. In the simple model, C_t and K_t obtained in the first period become C_{t-1} and K_{t-1} with respect to the next. Successive solutions trace out a path over time for all the endogenous variables.

Although this article focuses on the use of econometric models for forecasting purposes, the policy use of a model is illustrated here. In the simple model, there is only one variable that can be regarded as an instrument of government policy, namely government expenditures. It is necessary only to introduce into the model an alternative contemplated value for such expenditures under the assumed new policy and to solve the model under the changed conditions. The difference in the model's behavior under the two assumed values of government expenditures represents the effect of the proposed change.

By slightly enlarging the model, it is possible to illustrate another policy use. If the first equation is modified by substituting disposable income—income minus taxes—for total income and including an additional equation for taxes, the system is again complete with seven equations and seven unknowns. The model could then be used to examine the probable effects of a proposed change in tax rates. This would involve changing the parameters of the tax equation to conform with the proposed changes in rates and solving the model using the alternative tax functions.

The working of a simple model

At this stage, an attempt will be made to describe verbally how the illustrative model would work if it were used to forecast the effects of a given increase in government expenditures.

In the case of simple models, such a verbal account is possible, and it helps nonmathematicians to understand the essence of econometric models. In the case of models as complex as the OBE model that will be described, a verbal account is not possible.

1. The assumed increase in government expenditures will result in an increase in product (income) (equation 4). This, in turn, will result in an increase in consumption (equation 1), and this, in turn, in an increase in product (income) (equation 4), and so on, all within the same time period.

2. The assumed increase in government expenditures will also result in an increase in the profit component of income (equations 4 and 5), and this will stimulate investment (equation 2). Next, the increase in investment will affect production, income, and its profits component, and this will in turn stimulate investment (see the same equations). A profit-investment interaction will be in progress, similar to the income-consumption interaction sketched in paragraph 1.

3. The increases in investment, by raising income will also contribute to the income-consumption interaction described in paragraph 1; and the income-consumption interaction will contribute to the profit-investment interaction described in paragraph 2.

Thus, the initial increase in government expenditures will result in a cumulative upward movement in production and income and their components—consumption and investment and wages and profits. How far this cumulative movement will proceed depends on the spending behavior of consumers and investors. The higher the additional spending out of additional income, the larger the total effect of the initial increase in government expenditures. However, it can be shown that the upward movement will always reach a limit provided not all the additional income is spent.

This exhausts the effects of the increase in government spending on economic activity in the same period. However, there are additional effects in the next period.

4. In that period, consumption will increase further, reflecting the depend-

ence of current consumption on prior-period consumption (equation 1), and this will in turn tend to stimulate aggregate economic activity and its components in a manner very similar to that already sketched for the prior period.

5. However, another force will be working in the opposite direction: Investment during the prior period will have increased the capital stock, and this will reduce investment during the current period (equation 2). This will tend to bring about a cumulative downward movement in economic activity and its components.

Whether, how soon, and where the system will finally settle in response to the increase in government expenditures will depend on the initial state of the economy and the particular behavior patterns reflected in the equations. If the system does settle down to a unique income value, one may regard the effect of the additional government expenditure as the resulting (ultimate) change in output. The ratio of the change in output to the initial change in expenditure is called the *long-run multiplier*.³ If the ratio is computed on the basis of the first period effect only, it is called the *impact multiplier*. In a later section of this article, the impact multiplier for the OBE model will be given.

The above explanation of how the model works within a period illustrates the economic meaning of simultaneity. Mathematically, this is reflected in the fact that none of the equations can be used alone to solve for the left-hand variable; the system must be solved as a whole.

It would be possible by different specifications of equations to remove the simultaneous character of the simple model. We could, for example, substitute Y_{t-1} for Y_t in the first equation. Consumption would then depend exclusively upon lagged variables. In

3. In some contexts, the multiplier is confined to the effects on output of changes in exogenous variables operating through the consumption-income interrelationship. In this article, the use of the term is extended to include effects on output operating through the entire model. It should also be noted that a model does not have a single value for the multiplier. Different exogenous elements may have different effects. Thus, an assumed change in transfer payments would have a smaller effect on output than an equal change in purchases.

that case, the equation could be solved in isolation from the others since all values on the right would be known.

If the time period t is short enough, say a week, the substitution of lagged income for current income is not unreasonable; decisions to spend this week may well depend on last week's income and not on the current week's. When the time period is much longer—a quarter or more, as it is in almost all models—unidirectional causality becomes doubtful. That is, income earned within the quarter can clearly affect expenditures within the same period, so that causation runs in both directions. Such interdependence also applies to other variables and points up the importance of simultaneity in a realistic characterization of economic behavior.

Forecasting errors

Needless to say, econometric models do not produce perfect forecasts of the future. There are several reasons for this. First, errors can be made in the projections of the exogenous variables. In our simple example, for instance, government expenditures may turn out to be different from those that had been projected. Second, the data to which the equations are fitted usually contain errors; these will affect the estimates of the parameters. Incidentally, errors in the data will also result in a somewhat false standard against which errors of prediction are measured.

These two sources of error should be distinguished from those that occur in the construction and solution of the model and that would lead to faulty forecasts even if the exogenous variables and the data were perfect. To focus on these "model" errors, it is useful to regard an econometric model as a device that translates given inputs—the predetermined variables—into certain outputs, and to inquire into the reasons why this translation process may go wrong.

One reason for a model's failure to serve as a perfect translator stems from the fact that no conceivable set of equations can take full account of all the causal factors that influence given variables. We have already referred to the disturbance terms, which reflect the factors not taken into account in

the systematic parts of the equations. Although the assumption is made that the expected value of the disturbance terms is zero, in any given instance the actual value may be either positive or negative. This will result in differences between predicted and actual values.

A second type of error also is due to the disturbances; their presence tends to obscure underlying relationships, thus resulting in imprecise estimates of parameters. In other words, the parameter estimates are subject to sampling error because any given set of observations has associated with it a unique set of disturbances that would, in general, be different if the same structure underlay another set of observations.

Third, the various behavioral equations may not correctly specify the underlying economic relationships. In terms of our simple model, for instance, consumption may depend not only on current income and lagged consumption but also on, say, liquid assets held by consumers. This is likely to result in incorrect estimates of parameters and also in nonrandom residuals.

A final class of errors that may be distinguished stems from shortcomings in our methods of statistical inference. For instance, when two or more variables on the right-hand side of an equation tend to move closely together,

it is difficult to calculate their separate effects on the left-hand term. This again affects the parameter estimates. Also in this class is the problem of bias in the parameter estimates when the equations are part of a simultaneous system. (Appendix B contains a description of the methods used to cope with this problem in the present model.)

The reader might infer from the above listing that econometric models are beset with errors. This is far from true, as the subsequent discussion of the performance of the OBE model will show. The econometric approach is comparable in validity to alternative approaches—for instance, the "judgmental" method, which may also use econometric methods but which does not rely on an explicit set of simultaneous equations, or the "economic indicators" approach originally developed by the National Bureau of Economic Research. The particular promise of the econometric method stems from the fact that it provides explicit formulations of the cause-effect relationships in the economy which can be communicated and which are open to inspection and testing. In addition, compared with methods confined to predicting only directional change, the method has the clear advantage of quantification.

A Description of the OBE Model

The equations of the model presently in use at OBE are shown in Appendix A. This model represents the current stage in a process of development that began with the Wharton School model referred to in the introduction.

The original model, with only slight modification and with prices assumed exogenous, was tested at OBE over a fairly long period. During this period, certain changes were made.⁴ The model presented in this article incorporates all changes made up to the time of this writing. As research progresses and as changes in the economy warrant, further modifications will be made.

In its present form, the model consists of 49 equations including identities.

This section briefly describes the equations of the model and points out the principal mechanisms that merge the different parts into an interdependent system.

Categories of Equations

The model may conveniently be divided into six groups of equations: those explaining (1) components of

4. Some of the changes led to fairly important modifications of the original version, while others entailed relatively minor respecification. The most fundamental changes were the substitution of an explicit short-term labor demand function for an implicit relationship involving a production function, the introduction of an explicit equation for the overall price deflator, the substitution of a different equation for corporate profits, the further disaggregation of consumer durables, the introduction of an equation for housing starts, and the incorporation of a variable statistical discrepancy in the income-product identity.

GNP, (2) prices and wage rates, (3) labor force and employment-related magnitudes, (4) income components, (5) monetary variables, and (6) miscellaneous variables needed to round out the model. Each of these blocks of equations will be discussed briefly.

Components of GNP

Four equations explain personal consumption expenditures in 1958 dollars. These equations pertain to expenditures for autos and parts, other consumer durables, nondurables, and services other than housing. Housing services are projected exogenously. Each of the consumption components is made a function of disposable personal income, deflated by an appropriate price deflator, and of other relevant variables. Among the latter, lagged consumption, reflecting time taken to adjust consumption to changing income levels, figures prominently in the nondurables and services equations. Other relevant variables include the ratio of nonwage to wage income—which is introduced to allow for an income distribution effect—population, and deflated liquid assets held by households at the end of the preceding quarter.

Gross private domestic investment in 1958 dollars is estimated in three components: residential structures, fixed nonresidential investment, and the change in business inventories. For the residential component, an equation is included to predict the number of private nonfarm single-family housing units started during the quarter.⁵ Multifamily starts, which have become quantitatively significant only in recent years, are added exogenously because a satisfactory equation for them has not yet been developed. Expenditures on new nonfarm housing construction are obtained by multiplying the predicted starts by cost per unit started, expressed in 1958 dollars; this product is phased out over time by using a pattern developed by the Census Bureau. The total resi-

dential structures component is obtained by adding "additions and alterations" and investment in farm residential structures as exogenous variables.

Investment in nonresidential structures and producers' durable equipment depends primarily on businessmen's quarterly anticipations of plant and equipment expenditures reported in the OBE-SEC survey, converted into 1958 dollars. First anticipations—projections usually made 6 months in advance—are used in the equation. In addition to this variable, the equation contains some others, reflecting the factors that may cause actual investment to differ from anticipated investment. Such equations are frequently called realization equations.

The use of anticipatory data in a model, when such data are shown to be reliable, may be definitely advantageous for forecasting. However, the use of such data limits the time period over which forecasts can be made. For more extended forecasts, it would be necessary to substitute an equation reflecting the basic determinants of actual investment outlays for the equation containing the anticipatory data. Alternatively, supplementary equations designed to predict investment anticipations could be introduced.

For purposes other than forecasting, equations containing exogenous anticipatory variables are generally unsatisfactory. For instance, if one wishes to test the effects of alternative tax policies, the use in the model of exogenous investment anticipations is an obstacle, because it is not possible to determine the effect of the alternative policies on the anticipations.

Inventory investment is explained by total sales of private GNP to final markets, the prior period's inventory investment, durable manufacturers' unfilled orders, and total inventories on hand at the beginning of the period, all in 1958 dollars. The last variable, appearing with a negative coefficient, introduces a cycle-producing element into the model, as growth of inventories in the current period tends to dampen inventory investment in subsequent periods.

Imports (in 1958 dollars) are esti-

mated by two equations, one for finished goods and services and the other for crude materials and foodstuffs. The first is similar to the consumption functions in that it includes disposable income deflated by the implicit price deflator for imports and the ratio of nonwage to wage income. The materials and foodstuffs equation contains lagged private GNP divided by the import deflator.

Exports and government purchases of goods and services—both exogenous variables—complete the accounting for GNP.

Price and wage rate equations

Price indexes are needed to derive current-dollar estimates of GNP components and for other purposes, such as deflating disposable income or output in the various equations. Most indexes represent the appropriate implicit GNP deflators.

The equation for the price deflator for private GNP is a function of the average unit wage cost of private output for the current quarter and two previous ones, and of the two-quarter change in private final sales. The latter variable is made dependent upon capacity utilization in order to reflect increased sensitivity of prices to demand pressures when output is near capacity.

Three component deflators—those for consumer nonauto durables, nondurables, and fixed nonresidential investment—are made functions of the change in the overall price deflator and their own lagged values. Two other deflators—for consumer services and for residential structures—are made functions of the average wage rate. Deflators for autos and parts and for imports are exogenous.

The average (private sector) wage rate, which is estimated in the form of a percentage change over the previous four quarters, is related to the state of the labor market as measured by the unemployment rate during the intervening period, and to two factors that have a major role in collective bargaining decisions: changes in consumer prices and corporate profits. The relative wage change one year earlier—the change from eight to four quarters

5. This equation is a modified version of the one developed for total private housing starts by S. J. Mabel, "A Theory of Fluctuations in Residential Construction Starts," *American Economic Review*, June 1963, pp. 359-368. The rationale for the modified equation is discussed by Albert A. Hirsch, in "Predicting Housing Starts: Professor Mabel's Model Modified" (U.S. Department of Commerce, Staff Working Paper in Economics and Statistics No. 5, unpublished).

earlier—is also introduced. This term appears with a negative sign, suggesting that current wage changes are moderated by prior wage changes.

Labor force and employment equations

The labor force has increased secularly, both in absolute terms and as a proportion of the working-age population. It is also somewhat responsive to cyclical variations in employment. The labor force equation incorporates all of these elements. The dependent variable is expressed as a participation rate, and the explanatory variables are the proportion of the working-age population employed and a time trend.

Man-hours of labor employed are estimated in an equation reflecting both secular and cyclical variations in productivity.⁶ The secular variable is capacity output, which determines man-hour requirements at full capacity. Two other variables serve to adjust man-hours from full capacity to actual levels of production. One represents an intermediate adjustment of man-hours to an output level equal to a moving average of recently experienced output levels, called "planned" output. The other is a short-run adjustment to account for the difference between actual and planned output. Secular changes in man-hour requirements due to technological change, the growth of the stock of capital, and other factors are introduced by making two of the coefficients in the equations dependent upon time. For purely statistical reasons the equation was estimated by first dividing through by capacity output.

Private employment is derived by dividing the estimate of total man-hours by an index of average weekly hours worked. The equation for average hours worked contains the variables "capacity utilization" and "time" to reflect cyclical and secular movements.

Income equations

Income components represented by separate equations are: wages and salaries (including other labor income),

nonwage personal income (consisting of proprietors' income, rental income of persons, dividends, and personal interest income), corporate profits (including the inventory valuation adjustment), and dividends.

Private wages and salaries are obtained as the product of private man-hours and the wage rate (including other labor income); government employee compensation is estimated exogenously. The equation for corporate profits reflects the fact that profits are the excess of sales revenues over costs. Thus, corporate profits are made to vary positively with corporate sales and negatively with the ratio of the money wage rate to the overall price deflator, man-hours per unit of output, and the ratio of capacity to actual output. The last variable serves as a proxy for unit fixed costs.

Nonwage personal income less dividends is made a function of corporate profits and time. Corporate profits are introduced to reflect some association between the entrepreneurial income component and profits. The time trend is largely associated with the secular behavior of the other elements. Dividends are related to their value in the previous period and are also made to vary with current corporate profits.

Disposable personal income is obtained by adding total transfer payments to wage and nonwage incomes and subtracting personal tax and nontax payments and personal contributions for social insurance. Transfers other than unemployment compensation are exogenous.

Monetary equations

The model contains a small group of equations pertaining to monetary magnitudes. The short-term interest rate is made a function of excess reserves in the prior period and of the current rediscount rate. Both are exogenous to the model. The long-term rate is, in turn, made a function of the short-term rate and its own lagged value. The long-term rate is used in the equations for the FHA mortgage yield and for household liquid assets. The latter is also made to depend upon personal consumption expenditures to reflect transactions demand for money.

Miscellaneous equations

Finally, there are some equations that are not conveniently categorized. These are equations for new orders, unfilled orders, shipments, depreciation, unemployment compensation, personal tax and nontax payments, indirect business taxes, corporate tax liability, capacity output, and a number of identities required to complete the structure. Only brief mention will be made here of the more important functions.

New orders placed with manufacturers of durable goods are estimated by relating them to corporate profits. New orders, in turn, enter into the equation for shipments of these goods. The timing relationship between orders and shipments is variable and depends on the size of the lagged ratios of backlogs of unfilled orders to shipments.⁷ Unfilled orders, which are required also in the inventory investment and non-auto durables price deflator equations, are obtained from lagged unfilled orders and the difference between new orders and shipments. The new and unfilled orders, shipments, and corporate profits variables in the above relationships are deflated by an index of wholesale prices for durable goods.

Private output at capacity levels, used in a number of equations, is given by a production function relating output to labor and capital and an exponential trend to reflect technological advance. The equation has the Cobb-Douglas form and uses fixed nonresidential capital stock and 97 percent of the civilian labor force less government employment as measures of available capital and labor respectively.

The equation for personal tax and nontax payments is a simple relation between such payments and the sum of wage and salary and personal nonwage income. Indirect business taxes are related to final sales of private GNP and to time.

Of the many identities in the model, the one relating the income and product sides of the national income and product account deserves brief mention.

6. See Thomas A. Wilson and Otto Eckstein, "Short-Run Productivity Behavior in U.S. Manufacturing," *Review of Economics and Statistics*, February 1964, pp. 41-64.

7. The equation used for shipments has the same form as that used by Joel Popkin in "The Relationship Between New Orders and Shipments: An Analysis of the Machinery and Equipment Industries," *SURVEY OF CURRENT BUSINESS*, March 1965, pp. 24-32.

In addition to income and product flows, this statement contains the reconciliation items, which include the statistical discrepancy. In the present model, the discrepancy is not assumed at some predetermined value but is allowed to vary within certain limits imposed on its movement and level.⁸

The Model as an Interdependent System

The foregoing description of the equations does not make clear the interdependent character of the system. As noted in the discussion of interdependencies in the simple illustrative model, it is impossible to give an effective verbal account of the interdependence in a model consisting of many equations. However, with the aid of the flow chart

8. See Appendix C for the reasons for this treatment and an account of the constraints imposed.

(see chart 7), which depicts a simplified version of OBE's model, some idea may be obtained of the main interrelationships.

The rectangular boxes in the center of the chart represent, in condensed form, the main current endogenous variables in the model—the variables for which a simultaneous solution is sought. The rounded boxes to the left and right of the vertical dashed lines represent, respectively, the more important exogenous and lagged endogenous variables.

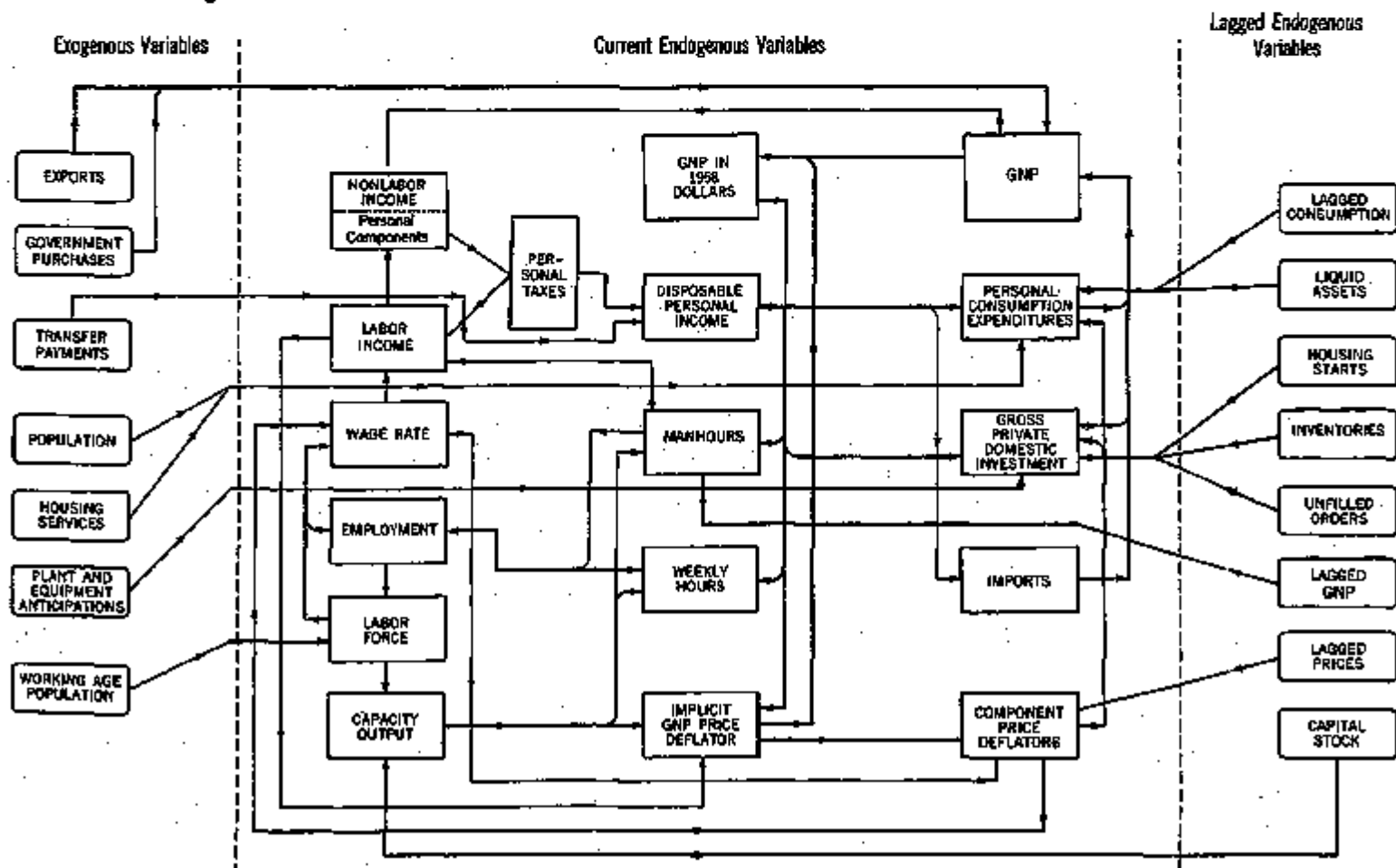
The important simplifications to note are: Compensation of government employees (GNP originating in government) is assumed to be zero. Consumption, investment, and import components have been aggregated into single variables. Component price deflators

are represented by one box. Corporate profits and personal nonwage income are consolidated into one nonlabor income variable, which is treated residually in the simplified version although not in the full model. Some relationships, such as those that determine unfilled orders, liquid assets, and housing starts, are not shown. The time variable, which appears in several equations, is left out, as are relatively minor explanatory variables. Finally, reconciliation items between national income and product are neglected.

The lines connecting the boxes of the chart reveal the direct dependencies among variables. The arrows indicate the cause-effect direction of these dependencies. In the chart, no distinction is made between behavioral equations and identities.

CHART 7

Condensed Flow Diagram



Some of the interrelationships in the system can now be traced. It is useful to point out first the linkage between product and income in the model. The boxes representing the GNP components at the right of the endogenous portion of the chart plus government purchases and exports make up total GNP. By deflating the latter (see the line connecting the implicit GNP deflator with the line emanating from GNP), GNP in 1958 dollars is obtained. The main linkage to the income side of the accounts is shown by the line leading from GNP in 1958 dollars to the box for man-hours and the box for weekly hours. One important link thus occurs via employment variables. The next of boxes concerned with employment and with the wage rate determines labor income. As was indicated earlier, non-labor income is determined residually in this simplified version of the model, that is, as the difference between GNP and labor income.

The feedback from income to product can also be delineated. As expected, the main linkage is revealed via the chain "income-taxes-disposable income-consumer expenditures." This chain can easily be followed in the chart.

The way in which prices are determined in the model can also be set forth. It is best seen by tracing the lines that lead into the implicit price deflator box. One such path emanates from GNP in 1958 dollars, another from labor income, and a third from capacity output. The first two of these flows combine to influence prices by changing unit labor costs. The first and third variables indicate the effect of capacity pressures on prices.

The description of the model given previously indicated that component prices are made functions of the overall implicit price deflator and, in some instances, of the wage rate. The main influence on component prices stems from the former—the box immediately adjacent—but it can also be seen that a line emanates from the wage rate and from lagged prices.

A number of other relationships can be followed in the chart. For example the relationships among the boxes concerned with employment and related variables can be traced. Em-

ployment is derived from man-hours and average weekly hours: To show this, a line from weekly hours joins one from man-hours and leads to employment. The wage rate is affected by unemployment—the difference between labor force and employment—and by prices. Thus, lines flow to the wage rate box from employment, labor force, and the component price deflators.

The reader will note that, with the exception of the rounded boxes representing the predetermined variables, which lie at the extreme right and left of the chart, all boxes have arrows entering them as well as emanating from them. This reveals the simultaneous character of the system and makes it possible to trace paths which are closed—that is, paths from any endogenous variable through other endogenous variables and back to the original variable. There are many such closed paths—or loops—in the system. The income-product loop is seen to be the main element of simultaneity.

Another important loop is that involving wages and prices.

The earlier discussion of the illustrative model introduced the concepts of long-run and impact multipliers. These ratios constitute important characteristics of specific models. In the present model, the multiplier is not a constant but depends to some degree on the levels of some variables. A test for a recent period yielded an impact multiplier on purchases of approximately 1.8. This means that if government purchases were to be changed by \$1.0 billion, the effect on output in the same quarter would be \$1.8 billion. Owing to the feedbacks via lagged endogenous variables, the cumulative effect would be larger in subsequent quarters. No figure is given here for the longrun multiplier because the present model neglects effects of changes in exogenous variables on the plant and equipment anticipations variable—an omission that would lead to an underestimate of long-run effects.

Testing the Model

Whether a model is to be used for forecasting or for studying policy or business cycles, the criterion of accepta-

bility must be the accuracy of the predictions it produces. In policy studies, in which interest focuses on quantita-

Table 1.—Predicted and Actual Gross National Product, 1953-66

(Billions of dollars seasonally adjusted at annual rates)

		Current Dollar Totals													
		1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	
1st Qtr.															
Predicted.....		356.6	362.5	353.1	408.9	442.3	426.8	473.5	505.7	510.7	548.8	572.8	613.9	658.1	
Actual.....		354.3	360.7	350.2	410.6	446.9	434.7	474.9	503.0	503.6	547.8	577.0	614.0	657.6	
2d Qtr.															
Predicted.....		363.2	368.2	356.0	415.1	445.5	442.0	485.5	505.1	518.3	540.5	582.0	628.3	670.6	
Actual.....		357.5	360.4	354.4	415.2	439.9	438.3	480.9	504.7	514.9	537.2	585.1	624.2	668.8	
3d Qtr.															
Predicted.....		373.9	380.1	407.1	415.3	439.4	445.0	483.6	506.6	522.9	538.7	562.3	605.5	653.3	
Actual.....		365.8	384.7	402.5	420.6	446.3	451.4	484.0	504.2	524.2	538.2	564.4	598.1	651.5	
4th Qtr.															
Predicted.....		372.6	374.4	412.2	427.4	435.9	459.3	487.4	508.7	543.3	575.8	604.3	635.1	697.7	
Actual.....		360.8	373.4	408.6	429.5	441.5	464.4	490.5	502.2	537.7	572.0	603.6	641.1	697.2	
Year															
Predicted.....		370.6	368.0	394.5	416.9	441.3	448.6	482.5	506.8	528.6	563.5	598.9	629.2	677.4	
Actual.....		354.5	354.6	378.0	419.2	441.1	447.3	483.6	503.8	523.1	560.3	589.7	628.7	676.3	
Year-to-year changes															
(Billions of current dollars)															
Predicted.....		25.1	3.4	25.0	15.9	22.1	5.0	34.2	23.2	21.8	41.4	27.7	40.0	45.7	
Actual.....		19.1	0.2	33.2	21.2	21.9	0.2	30.3	20.3	16.3	41.2	25.9	39.5	47.5	
(Billions of 1958 dollars)															
Predicted.....		21.4	-0.6	24.0	7.6	10.5	-4.2	31.5	14.9	14.2	34.3	17.1	27.1	29.3	
Actual.....		17.7	-5.8	31.0	8.0	6.6	-5.2	23.0	11.9	9.5	32.7	20.0	27.6	32.0	

Source: U.S. Department of Commerce, Office of Business Economics.

tive differences in economic behavior resulting from alternative policy actions, it is necessary, as was noted earlier, to take all major policy instruments into account and to derive endogenously as many as possible of the nonpolicy variables. This may result in some loss of forecasting accuracy. But even in policy applications, forecasting accuracy must be reasonably good if one is to have confidence that the dynamic structure of the economy has been adequately captured by the set of equations.

This section presents three sets of results: (1) a quantitative analysis of the overall behavior of the model during the entire period 1953 through 1965; (2) an examination of the model's performance in predicting cyclical turning points; and (3) a detailed presentation of the model's performance for 1965, a year that lies outside the period over which the equations were fitted.

These results do not represent forecasts in the usual sense of prediction of events before they occur. They are, rather, *ex-post* forecasts in which exogenous variables are assigned their actual values. Lagged endogenous variables, however, are those generated by the model as current endogenous variables of prior quarters. While such tests are not strictly pertinent to an actual forecasting situation, they have the advantage of eliminating errors made in projecting the exogenous variables. Obviously, in judging the validity of a model, errors due to wrong assumptions about the exogenous variables are not relevant.

There is, however, a sense in which tests for the period prior to 1965 are not fully adequate. Since this is the period to which the equations of the model were fitted, it is somewhat uncertain whether the basic structure of economic behavior was captured or whether the equations reflect special factors unique to the period. There is the further point that the structure of the economy may have changed since the period over which the equations were fitted. The only conclusive test of forecasting accuracy is whether a model continues to perform satisfactorily beyond the period from which it was derived. This limitation, however,

does not imply that *ex-post* forecasts are of no value. Adequate performance over the fitted period is at least a necessary condition for acceptance; a model that performs poorly over the fitting period is not likely to be a good forecasting tool.

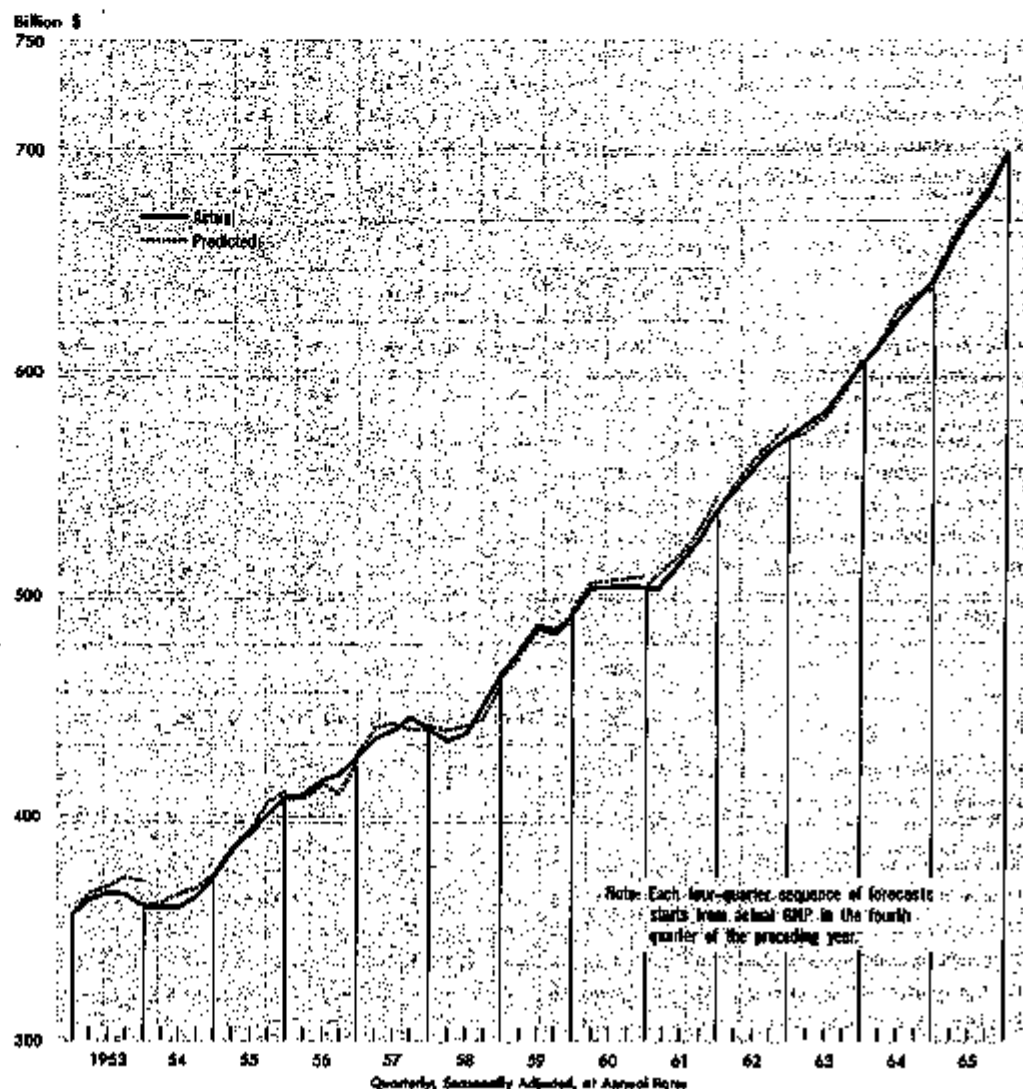
It is important to note in this connection that apart from the tests of the individual equations discussed earlier, the model requires testing as a whole. Even if the separate equations fit well, have statistically significant coefficients, and are theoretically reasonable, the model as a whole may still perform unsatisfactorily. This may be because

the simultaneous solution of the entire system and the use of an earlier period's outputs as later inputs may cause errors.

Model Performance, 1953-65

To test the model's quantitative performance, *ex-post* forecasts of economic activity were made for each of the 13 years 1953 through 1965. In each case, the model was run for the four quarters of the year using the fourth quarter of the previous year as the jumpoff point. Known values of exogenous variables were used through-

Predicted Versus Actual GNP, 1953-65



out. All lagged endogenous variables arising from quarters within the year were those yielded by the model rather than actual values. Thus, the results provide a test of how accurately the model generates a sequence of outputs from an initial starting point.

Major results of the tests are shown in tables 1, 2, and 3 and in chart 8. Table 1 gives predicted and actual values of GNP in current dollars by quarter and by year. The last two pairs of lines show predicted and actual

year-to-year changes in current- and constant-dollar GNP. Table 2 lists the errors in predicting current-dollar GNP and its major components, disposable personal income, real GNP, and the implicit price deflator for GNP. Errors are defined as predicted minus actual values. Table 3 presents summary statistics on errors for the same items. The chart shows predicted and actual GNP; each four-quarter forecast is shown as starting from its prior fourth quarter actual GNP jumpoff.

General time path

Table 1 and the chart show that the model performed quite well over the period. For 9 of the 13 years, the error in predicting GNP for the year was \$3.0 billion or less. As shown in table 3, the average absolute error (obtained by disregarding the signs of the individual errors) for all 13 forecasts was \$2.3 billion. The average absolute error for constant-dollar GNP was \$2.9 billion. As the bottom line in table 1

Table 2.—Quarterly and Annual Prediction Errors: Selected Items, 1953-65

(Billions of current dollars seasonally adjusted at annual rates, unless not applicable)

	1953					1954					1955					1956				
	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year
Gross national product.....	2.4	1.7	8.1	11.8	8.0	1.8	5.8	4.4	1.0	3.2	-1.1	0.6	4.6	3.4	1.8	-1.7	-0.1	-5.3	-2.1	-2.3
Personal consumption expenditures.....	.5	3.2	7.2	11.9	5.6	2.9	2.9	2.3	1.2	2.3	-1.2	.6	4.2	4.6	2.0	.3	1.3	-1.3	1.0	.2
Residential structures.....	-8	-1.0	-3	.9	-6	-2	-5	-9	-7	-5	-1	.2	.6	1.4	.6	-1.2	-2.0	-1.7	-1.0	-1.5
Fixed investment, nonresidential.....	.9	-5	-4	.1	0	.6	1.5	1.0	1.3	1.2	.3	9	-8	-1.2	-6	-4	1.7	.6	.7	.7
Change in business inventories.....	2.0	.5	2.6	2.0	1.9	-5	1.8	2.5	-4	.9	-5	-3	.6	-1.4	-4	-6	-1.2	-3.2	-2.7	-2.9
Net exports.....	.0	-5	-9	-2.1	-9	-1.1	.3	-6	-7	-5	.4	.1	-1	.2	.3	.6	.1	.9	-1	.3
Disposable personal income.....	-3	.7	5.9	9.2	3.8	1.4	4.9	4.9	3.8	3.8	.7	1.6	3.0	4.2	2.3	-1.1	-6	-4.0	-3.6	-2.3
GNP in constant (1958) dollars.....	2.0	.7	7.4	4.7	3.7	5.0	8.6	5.6	.2	4.9	.5	1.5	6.2	3.9	3.0	.5	.9	-3.1	.2	-1.1
Implicit price deflator for GNP (1958=100).....	.1	.2	.4	1.7	.7	-6	-5	-1	.3	-3	-3	-2	-2	0	-2	-5	-2	-5	-6	-4
	1957					1958					1959					1960				
	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year
Gross national product.....	5.4	3.6	-6.9	-1.6	0.2	3.1	4.3	-6.4	-5.2	-7	-6.5	-1.3	-6.4	-3.1	-1.1	2.7	1.4	2.4	5.4	3.0
Personal consumption expenditures.....	.2	1.4	-4.3	-3.4	-1.5	1.6	1.8	-1.9	-3	.3	-1.7	-2.6	-4.2	-4.8	-2.3	4.0	-6	1.7	2.7	2.0
Residential structures.....	1.7	.7	-4	-4	.4	.3	-4	-2.8	-3.3	-1.6	-1	.6	1.0	1.0	.7	-1.3	-7	-4	-4	-6
Fixed investment, nonresidential.....	.6	.2	-1.8	-1.6	-7	.4	-1	-3	.3	.1	-1	-1.4	-3	-3	-6	-3	.3	1.1	.4	.4
Change in business inventories.....	1.9	7.1	-1.4	2.7	1.1	2.7	2.7	-1.8	-2.9	.2	1.1	.9	1.5	-1	1.0	.2	1.8	.4	4.5	1.7
Net exports.....	.9	.4	.9	.9	.8	.2	.1	.4	1.1	.4	.3	1.2	1.5	1.3	1.1	.3	.6	-3	-1.3	-3
Disposable personal income.....	1.3	.7	-5.9	-2.8	-1.7	1.3	1.7	-5.4	-2.9	-1.3	-7	-1.7	.3	-3	-6	1.7	-1.3	-7	2.3	.6
GNP in constant (1958) dollars.....	5.7	5.6	.1	5.2	4.1	7.4	5.3	-4.9	-3.8	1.0	2.3	2.5	4.6	1.9	2.9	1.9	1.2	2.6	6.1	3.0
Implicit price deflator for GNP (1958=100).....	-1	-4	-1.5	-1.5	-9	-4	-2	-3	-3	-4	-6	-7	-1.1	-1.0	-8	.2	.1	.6	-2	.0
	1961					1962					1963					1964				
	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year
Gross national product.....	7.1	2.4	5.7	6.6	6.6	2.5	4.3	3.8	3.2	-3.2	-2.1	-6.3	1.2	-1.2	-0.1	4.1	1.7	-3.0	0.6	0.5
Personal consumption expenditures.....	5.4	4.6	6.3	4.6	4.9	-8	2.8	3.1	4.0	2.3	-2.7	-2	.4	5.2	.7	.7	3.2	.4	.7	1.3
Residential structures.....	-4	.4	.9	.3	.3	-2	-6	-3	-1	-3	-7	-4	.4	-4	-3	-6	.2	.5	.0	.1
Fixed investment, nonresidential.....	-1	-2	-2	.7	.2	.6	-7	-7	.4	-1	.6	-3	.3	.3	.2	-1.6	-2	-9	-2.5	-1.3
Change in business inventories.....	3.2	-7	-4	.4	.7	.1	1.4	2.2	-1	1.0	-3	-1.3	-2.1	-3.5	-1.7	1.7	1.0	1.8	-2.2	.4
Net exports.....	-1.1	-1.1	.1	-2	-5	.3	.5	.1	-2	.2	-1	.2	.1	-3	.1	-3	-2	.0	1.0	.1
Disposable personal income.....	3.0	.6	1.1	.2	1.3	.7	1.3	3.8	6.0	2.9	-2.3	-1.4	.0	1.0	-8	1.2	2.5	1.3	-2.5	.7
GNP in constant (1958) dollars.....	6.1	2.4	4.0	5.6	5.7	1.8	2.6	2.6	-2	1.6	-2.1	-1.6	-4.2	-3.8	-2.9	-6	.0	-5	-9	-5
Implicit price deflator for GNP (1958=100).....	.2	.1	.3	-3	.1	-3	.2	.3	.8	.8	-2	-1	.7	.9	.4	.1	.7	.8	-3	.1
	1965					1966					1967					1968				
	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year	1Q	2Q	3Q	4Q	Year
Gross national product.....	7.1	2.4	5.7	6.6	6.6	2.5	4.3	3.8	3.2	-3.2	-2.1	-6.3	1.2	-1.2	-0.1	4.1	1.7	-3.0	0.6	0.5
Personal consumption expenditures.....	5.4	4.6	6.3	4.6	4.9	-8	2.8	3.1	4.0	2.3	-2.7	-2	.4	5.2	.7	.7	3.2	.4	.7	1.3
Residential structures.....	-4	.4	.9	.3	.3	-2	-6	-3	-1	-3	-7	-4	.4	-4	-3	-6	.2	.5	.0	.1
Fixed investment, nonresidential.....	-1	-2	-2	.7	.2	.6	-7	-7	.4	-1	.6	-3	.3	.3	.2	-1.6	-2	-9	-2.5	-1.3
Change in business inventories.....	3.2	-7	-4	.4	.7	.1	1.4	2.2	-1	1.0	-3	-1.3	-2.1	-3.5	-1.7	1.7	1.0	1.8	-2.2	.4
Net exports.....	-1.1	-1.1	.1	-2	-5	.3	.5	.1	-2	.2	-1	.2	.1	-3	.1	-3	-2	.0	1.0	.1
Disposable personal income.....	3.0	.6	1.1	.2	1.3	.7	1.3	3.8	6.0	2.9	-2.3	-1.4	.0	1.0	-8	1.2	2.5	1.3	-2.5	.7
GNP in constant (1958) dollars.....	6.1	2.4	4.0	5.6	5.7	1.8	2.6	2.6	-2	1.6	-2.1	-1.6	-4.2	-3.8	-2.9	-6	.0	-5	-9	-5
Implicit price deflator for GNP (1958=100).....	.2	.1	.3	-3	.1	-3	.2	.3	.8	.8	-2	-1	.7	.9	.4	.1	.7	.8	-3	.1

NOTE.—Error equals predicted minus actual.

SOURCE: U.S. Department of Commerce, Office of Business Economics.

shows, the model predicted the declines in constant-dollar GNP in both 1954 and 1958.

These results are highly summary and conceal strengths and weaknesses in predicting quarterly economic behavior as well as the behavior of individual components. Table 3 shows, for example, that (average absolute) errors are not, in general, uniform throughout the year. For current-dollar GNP, the error tends to increase with the distance from the jumpoff quarter, although the pattern is not completely consistent. The error made in fourth quarter predictions, for example, was \$3.7 billion, as compared with \$2.5 billion for the first quarter. This is not surprising, since successive quarterly forecasts embody whatever errors were made in prior periods' components, and these enter as inputs in later periods.

In some instances, relatively small errors in GNP for the year as a whole reflect offsetting positive and negative errors made in the individual quarters. For 1957 as a whole, for example, predicted GNP differed from actual GNP by only \$0.2 billion, because an overestimate of \$4.5 billion for the first half of the year was virtually offset by an underestimate for the second half.

Quarterly errors in current-dollar GNP ranged from a low of -\$6.9 billion (third quarter of 1957) to a high of \$11.8 billion (fourth quarter of 1953); errors in constant-dollar GNP ranged from -\$7.2 billion (fourth quarter of 1965) to \$3.8 billion (second quarter of 1954). However, the summary measures given in table 3 show that such large errors were exceptional.

Absolute errors in components

Comparatively small errors in total GNP may also reflect larger but partly offsetting errors in the components, as can be seen from table 2. In general, however, errors in components were also moderate.

The largest errors occurred in consumption expenditures. Average absolute errors in this component were about the same as for total GNP. One might well expect this since consumption expenditures account for about two-thirds of GNP and usually for a large proportion of its changes.

Errors in predicting residential construction, fixed nonresidential investment, and net exports were relatively small. (Errors in net exports reflect errors in imports since exports are exogenous.) Average absolute errors in each of these items for all quarters and years were less than \$1.0 billion.

On the average, errors in inventory change were somewhat larger than those in the last three items mentioned but less than those in consumption expenditures. Errors in inventory change were often relatively large, but it should be

Table 3.—Summary Measures of Quarterly and Annual Prediction Errors for Selected Items, 1953-65

(Billions of current dollars seasonally adjusted at annual rates, unless not applicable)

	Average absolute error	Average error	Range	
			Low	High
Gross national product:				
1Q	2.5	1.5	-3.2	7.1
2Q	2.6	2.0	-2.1	5.8
3Q	4.1	1.0	-5.9	8.1
4Q	3.7	1.6	-5.2	11.8
Year	2.3	1.5	-2.3	6.0
Personal consumption expenditures:				
1Q	1.9	1.0	-2.7	5.4
2Q	2.0	1.4	-2.6	4.6
3Q	2.0	1.1	-4.3	7.2
4Q	3.5	2.3	-4.9	11.9
Year	2.2	1.4	-2.3	3.6
Residential structures:				
1Q	.7	-.4	-1.3	1.7
2Q	.6	-.5	-2.0	.7
3Q	.8	-.3	-2.8	1.0
4Q	.8	-.6	-3.3	1.4
Year	.6	-.2	-1.6	.7
Fixed investment, nonresidential:				
1Q	.6	.0	-1.6	.6
2Q	.6	.0	-1.4	1.7
3Q	.7	-.3	-1.8	1.1
4Q	.9	-.2	-2.6	1.3
Year	.6	-.1	-1.3	1.3
Change in business inventories:				
1Q	1.3	.5	-.8	3.3
2Q	1.2	.7	-1.2	2.7
3Q	1.6	.9	-3.3	2.6
4Q	2.0	-.5	-3.5	4.5
Year	1.1	.3	-2.0	1.1
Net exports:				
1Q	.4	.1	-1.1	.6
2Q	.6	.3	-1.1	2.1
3Q	.6	.3	-.9	2.0
4Q	.6	.1	-2.1	1.8
Year	.5	.2	-.9	1.4
Disposable personal income:				
1Q	1.4	.6	-2.8	2.0
2Q	1.5	.7	-1.7	4.9
3Q	3.0	.6	-5.9	8.9
4Q	3.3	1.4	-3.5	9.2
Year	1.7	.9	-2.3	3.9
GNP in constant (1958) dollars:				
1Q	2.9	2.3	-2.1	7.4
2Q	2.6	2.3	-1.6	6.6
3Q	2.7	1.5	-4.9	7.4
4Q	3.4	1.0	-7.2	6.6
Year	2.9	1.7	-2.9	4.9
Implicit price deflator for GNP (1958=100):				
1Q	.3	-.1	-.6	.4
2Q	.3	.0	-.7	.7
3Q	.5	-.1	-1.6	.7
4Q	.7	-.1	-1.5	1.7
Year	.4	-.1	-.3	.7

SOURCE: U.S. Department of Commerce, Office of Business Economics.

remembered that inventory change is the most volatile element in GNP.

Price behavior was perhaps the poorest aspect of the model results. Average absolute errors in the implicit GNP deflator were 0.3 points for each of the first two quarters, or only somewhat less than the average quarterly increase in the actual deflator; for the third and fourth quarters, the errors were larger. However, the equation system is such that errors in the price index and in real output tend in opposite directions; thus, current-dollar GNP does not bear the full brunt of errors in price.

Evidence of bias

There is evidence that errors of prediction in the model are not entirely random. For the period as a whole, there was a slight tendency to overestimate GNP. This is indicated by positive average errors (obtained by netting positive and negative errors), shown in the second column of table 3 for each quarter and for the year as a whole; somewhat larger average errors are observed for real than for current-dollar GNP.

The tendency to overestimate GNP reflected primarily a similar tendency in personal consumption expenditures. Table 2 shows that positive errors in consumption were generally associated with positive errors in disposable income—an important determinant of consumption. However, such errors were not perfectly correlated. Furthermore, disposable income exhibited smaller average errors than did consumption.

Average errors in GNP components other than consumption were all less than \$1.0 billion and in most cases less than \$0.5 billion, indicating little or no bias in estimating these components. Despite sizable average absolute errors in the implicit GNP deflator, there was no apparent bias in estimating it.

Business Cycle Turning Points

Tests of a model's performance in predicting business cycle turning points are clearly important in an overall appraisal. Success in making such predictions strongly suggests that criti-

cal dynamic elements in the economy have been taken into account in the set of equations. Failure to pass such tests reflects adversely on a model's reliability, at least for periods when economic activity is undergoing changes in direction.

Such tests can be applied with varying degrees of rigor. A stringent criterion of success is the requirement that all turning points be estimated with precise timing. This test is particularly rigorous when actual changes in direction are slight. An alternative criterion is that forecasts show a directional change in the neighborhood of the actual turning point. Although considerably less rigorous, such a criterion still permits appraisal of the model's usefulness since a somewhat mistimed signal of

change is clearly better than no signal at all.

In this section, the behavior of the model in predicting constant-dollar GNP at its six cyclical turning points during the 1953-61 period is examined. Three separate four-quarter forecasts were made preceding each turning point. The first used as a jumpoff the quarter three periods before the actual reversal; the second and third started, respectively, from two quarters and one quarter before the reversal. Thus, there were in all 18 forecasts, 9 for upturns and 9 for downturns.⁹

Chart 9 presents the forecasts of both constant-dollar and current-dollar GNP for each of the turning points. The

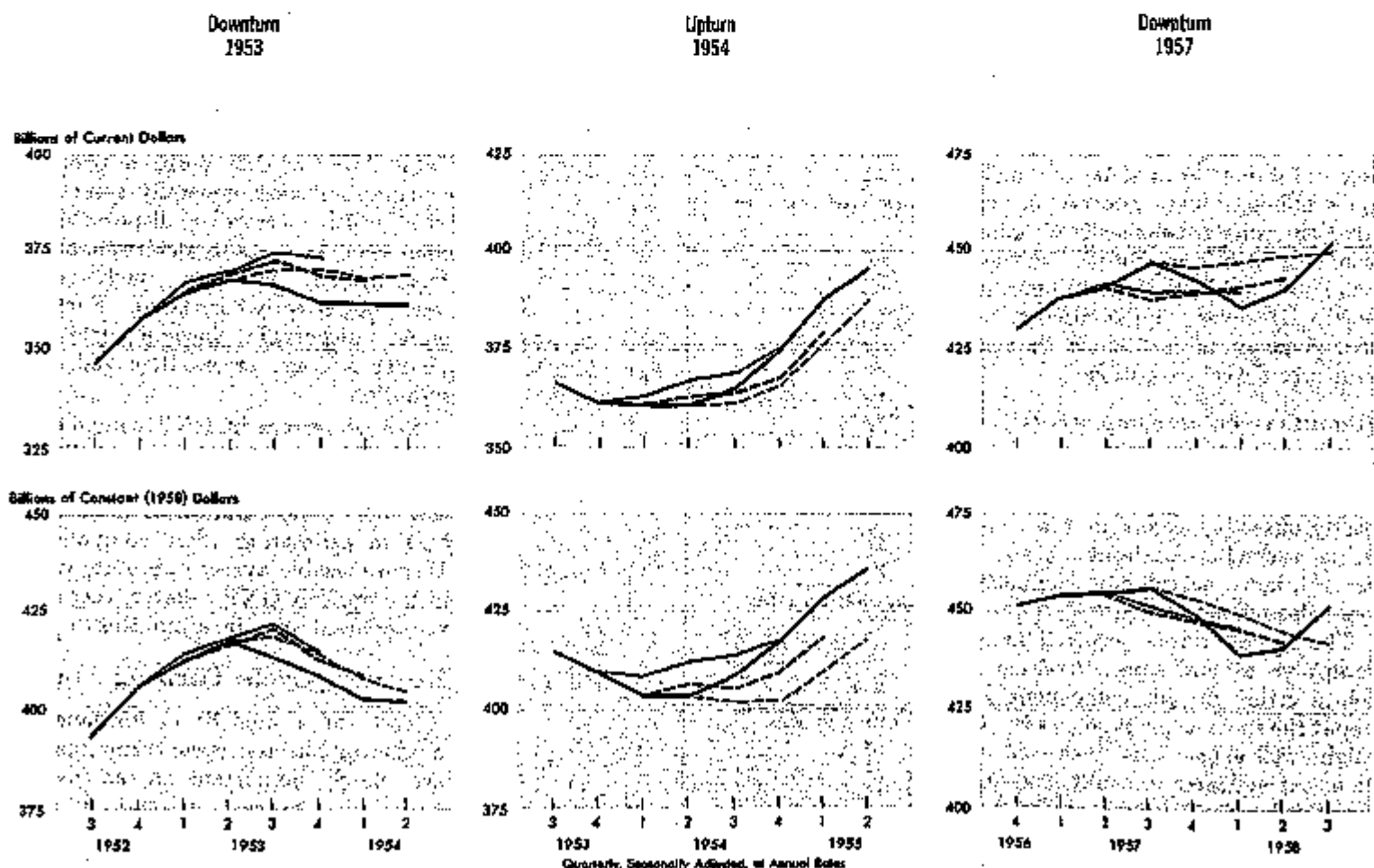
discussion focuses on constant-dollar GNP because it is the most comprehensive measure of real economic activity.

Summary of turning point behavior

The rigorous criterion of exactly coincident timing was met by the model only infrequently. Three of the nine forecasts of downturns were precisely timed—one made two quarters and two made one quarter in advance. None of the forecasts made three quarters ahead manifested precise timing. In recoveries, timing was accurate only when the forecast was made one quarter before the actual upturn; prediction was accurate in two of the three cases. The results at both peaks and troughs suggest that precision is in-

9. The forecast three quarters before the 1955 upturn jumped off from the peak quarter in 1957 and thus is also a forecast made one quarter in advance of the downturn that followed.

Current and Constant Dollar GNP at Cyclical Turning Points, 1953-61: Predicted Versus Actual



creased when the jumpoff quarter is close to the actual turning point.

The performance of the model was very good when the criterion was relaxed to require only that it predict a turning point in the neighborhood of the actual turning point—for instance, one quarter on either side. The chart shows that all but 3 of the 18 forecasts met this criterion. The exceptions were forecasts made three quarters before directional changes occurred.

The foregoing summary was concerned solely with the extent to which turning points were successfully predicted. The following section is a brief analysis of the model's behavior with particular reference to individual cycles.

Performance in individual cycles

Perhaps the best performance at cyclical turning points was in the

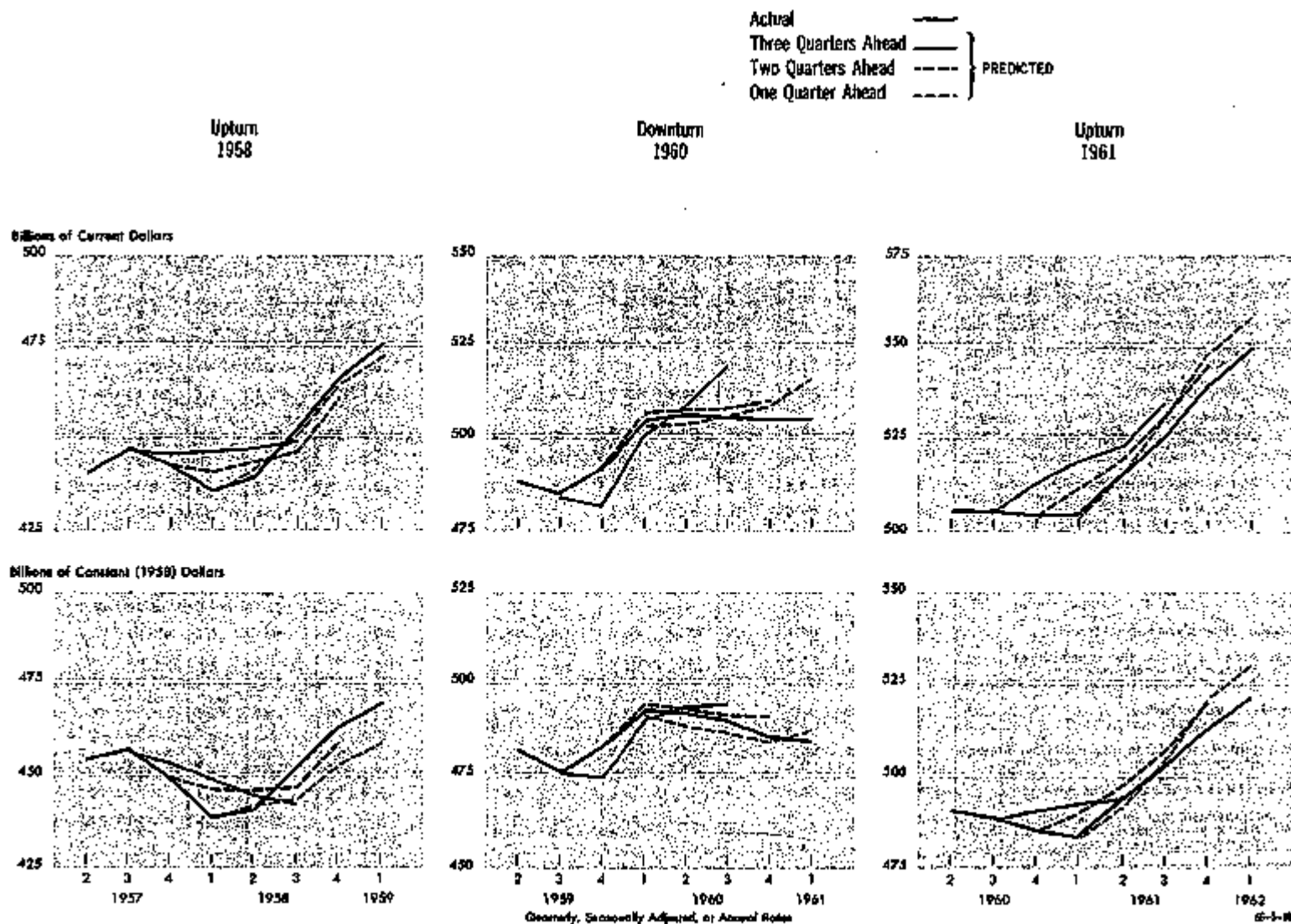
1957-58 period. Forecasts two and three quarters before the fourth quarter 1957 decline showed a contraction in activity in the third quarter. The forecast made one quarter before the actual turning point predicted it correctly. All three of these forecasts warned of a substantial decline in constant-dollar GNP, similar to that which actually occurred.

Beginning two quarters ahead, the model also predicted the 1958 upturn and to some extent its strength. Of particular interest is the forecast made two quarters before the upturn began. It shows a continuation in the decline of real GNP for one more quarter, followed by a leveling off prior to recovery. The forecast one quarter before the upturn correctly predicted the recovery.

On balance, the behavior of the model in the mild recession of 1960-61 was not as good as in the 1957-58 recession. The model performed as well, if not better, in predicting the downturn, but was markedly less successful in predicting the upturn.

With respect to the 1960 downturn, the forecast made three quarters earlier started from the third quarter of 1959. This quarter was dominated by the contractionary influence of a strike in the steel industry. The model predicted a continued decline for one quarter, a sharp advance for one quarter, and much smaller advances for the two quarters in which actual constant-dollar GNP was edging down from its peak. The forecast made two quarters before the downturn gave early warning of the exact quarter in which it would

CHART 9



start. The forecast made one quarter before the downturn was timed correctly and the predicted decline was about the right size. In addition, this forecast indicated the ensuing upturn, but placed it in the first quarter of 1961, when actual constant-dollar GNP fell only slightly further to reach its trough for that recession.

The prediction made three quarters before the upturn was quite inaccurate, forecasting the recovery two quarters before it actually occurred. The forecast made one quarter later also gave a

premature signal. The forecast made at the trough correctly indicated recovery. However, in view of the failure of the two preceding forecasts to materialize, it could easily have been discounted as another premature signal.

The forecasts for the 1953-54 period, particularly for the recovery, were least satisfactory though still relatively useful. All forecasts, including the one made three quarters ahead, showed a recession but in each case one quarter later than it actually occurred. Despite the timing error, the persistency with

which the model suggested a recession made the forecasts of value. During the recession, two premature signals of recovery were obtained, although the second one suggested it would be abortive. A continuation of the decline in constant-dollar GNP was forecast at the trough.

Forecast for 1965

As was pointed out earlier, the forecast for 1965, since it is outside the period to which the equations were

Table 4.—Predicted and Actual Gross National Product and Components, Income and Reconciliation Items, and Selected Supplementary Items, 1965

(Billions of dollars seasonally adjusted at annual rates, unless not applicable)

	1964- 4th Qtr.	1st Quarter				2d Quarter			3d Quarter			4th Quarter			Year		
	Actual	Predicted	Actual	Difference	Predicted	Actual	Difference	Predicted	Actual	Difference	Predicted	Actual	Difference	Predicted	Actual	Difference	
Gross national product.....	641.1	658.1	657.8	8.5	670.5	668.8	1.7	682.2	681.5	0.7	697.7	697.2	0.5	677.4	676.8	0.6	
Personal consumption expenditures.....	406.9	419.8	418.9	2.9	424.1	424.5	-0.4	434.4	432.5	1.9	444.0	441.0	3.0	430.6	429.7	0.9	
Automobiles and parts.....	24.8	28.9	30.3	-1.4	28.2	29.3	-1.1	29.2	30.3	-1.1	30.1	30.1	0.0	29.4	30.0	-0.6	
Durable goods other than automobiles and parts.....	33.1	34.5	34.2	0.3	35.0	34.2	0.8	36.1	35.0	1.1	36.0	36.3	-0.3	35.9	35.0	0.9	
Nondurable goods.....	180.9	185.0	182.8	2.2	187.5	187.9	-0.4	191.9	190.5	1.4	195.3	195.0	0.3	189.9	189.5	0.4	
Services.....	167.1	172.4	169.5	2.9	173.4	173.1	0.3	177.3	176.7	0.6	180.6	179.6	1.0	175.4	174.7	0.7	
Gross private domestic investment.....	97.7	109.7	103.4	-2.7	102.9	102.5	0.4	104.7	102.9	1.8	105.9	110.3	-4.4	103.4	106.7	-3.3	
Fixed investment, nonresidential.....	63.5	65.8	66.9	-1.1	66.6	65.4	1.2	69.7	70.9	-1.2	72.5	73.0	-0.5	69.1	69.8	-0.7	
Residential structures.....	26.7	26.8	27.7	-0.9	27.4	28.0	-0.6	27.5	27.7	-0.2	28.4	27.2	1.2	27.0	27.6	-0.6	
Change in business inventories.....	7.5	8.3	8.8	-0.5	8.9	8.4	0.5	7.0	7.6	-0.6	7.0	10.1	-3.1	7.3	8.2	-0.9	
Net exports of goods and services.....	8.9	8.5	6.0	2.5	10.1	9.0	1.1	9.4	7.4	2.0	8.8	8.9	-0.1	8.7	7.1	1.6	
Exports.....	33.4	34.7	34.7	0.0	40.4	40.4	0.0	40.1	40.1	0.0	40.8	40.8	0.0	39.0	39.0	0.0	
Imports.....	29.5	26.2	28.6	-2.4	30.3	32.4	-2.1	30.7	32.7	-2.0	32.0	33.9	-1.9	30.3	31.9	-1.6	
Government purchases of goods and services.....	128.6	131.3	131.3	0.0	133.5	133.5	0.0	135.4	135.4	0.0	139.0	139.0	0.0	134.6	134.6	0.0	
Gross national product.....	641.1	658.1	657.8	8.5	670.5	668.8	1.7	682.2	681.5	0.7	697.7	697.2	0.5	677.4	676.8	0.6	
Less: Capital consumption allowances.....	55.9	57.7	57.7	0.0	58.3	58.3	0.0	59.1	59.1	0.0	59.8	59.8	0.0	58.7	58.7	0.0	
Indirect business tax and nontax liability.....	59.3	60.9	61.6	-0.6	62.5	61.4	1.1	62.2	62.0	0.2	63.5	62.9	0.6	62.3	62.0	0.3	
Business transfer payments.....	2.4	2.3	2.3	0.1	2.3	2.3	0.0	2.3	2.3	0.0	2.3	2.3	0.0	2.3	2.3	0.0	
Statistical discrepancy.....	-2.2	-1.2	-2.1	0.9	-2.5	-1.4	1.1	-1.5	1.4	-2.9	1.0	2.4	-1.4	0.0	-2.3	2.3	0.0
Plus: Subsidies less current surplus of govern- ment enterprises.....	1.8	1.4	1.4	0.4	1.8	1.8	0.0	1.8	1.2	0.6	1.1	1.1	0.0	1.2	1.2	0.0	
Equals: National income.....	526.3	538.8	545.5	-6.7	549.0	549.5	-0.5	560.4	557.9	2.5	572.2	570.8	1.4	556.4	554.7	1.7	
Less: Corporate profits and inventory valuation adjustment.....	64.9	69.0	71.7	-2.7	70.8	72.0	-1.2	72.9	73.5	-0.6	72.2	75.3	-3.0	71.1	73.1	-2.0	
Contributions for social insurance.....	28.4	28.9	28.9	0.0	29.2	29.3	-0.1	29.6	29.6	0.0	30.2	30.2	0.0	29.5	29.5	0.0	
Wage accruals less disbursements.....	-1.1	0.0	0.0	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
Plus: State unemployment insurance benefits.....	2.4	2.3	2.4	-0.1	2.0	2.2	-0.2	2.0	2.2	-0.2	2.0	2.0	0.0	2.0	2.2	-0.2	
Business and other government transfer payments to persons.....	34.4	36.9	36.9	0.0	36.3	35.2	1.1	36.0	36.0	0.0	37.6	37.6	0.0	36.9	36.9	0.0	
Interest paid by government (net) and by consumers.....	19.5	19.9	19.0	0.9	20.4	20.4	0.0	20.5	20.5	0.0	21.1	21.1	0.0	20.6	20.6	0.0	
Dividends.....	17.7	18.2	18.0	0.2	18.7	18.6	0.1	19.2	19.2	0.0	19.7	19.9	-0.2	19.0	18.9	0.1	
Equals: Personal income.....	597.2	618.1	616.2	1.9	625.4	624.7	0.7	636.5	635.8	0.7	649.3	646.8	2.5	630.3	629.7	0.6	
Less: Personal tax and nontax payments.....	60.7	63.2	64.8	-1.6	66.5	66.2	0.3	65.1	64.8	0.3	65.9	65.7	0.2	65.7	65.4	0.3	
Equals: Disposable personal income.....	446.4	454.9	451.4	3.5	458.9	458.5	0.4	471.4	471.0	0.4	483.4	481.1	2.3	464.6	464.3	0.3	
Less: Personal consumption expenditures.....	406.9	419.8	418.9	2.9	424.1	424.5	-0.4	434.4	432.5	1.9	444.0	441.0	3.0	430.6	429.7	0.9	
Interest paid by consumers.....	10.4	10.5	10.6	-0.1	11.0	11.0	0.0	11.3	11.3	0.0	11.6	11.6	0.0	11.1	11.1	0.0	
Personal transfer payments to foreigners.....	0.6	0.6	0.6	0.0	0.6	0.6	0.0	0.6	0.6	0.0	0.6	0.6	0.0	0.6	0.6	0.0	
Equals: Personal saving.....	38.9	24.6	22.9	1.7	23.2	22.4	0.8	23.1	23.8	-0.7	23.2	27.1	-3.9	25.4	24.9	0.5	
Saving rate (percent).....	8.6	4.8	5.2	-0.4	5.1	4.9	0.2	5.0	5.7	-0.7	5.8	6.6	-0.8	5.4	5.4	0.0	
Gross national product in constant (1968) dollars.....	584.7	606.2	607.7	-1.5	603.0	603.5	-0.5	611.2	610.0	1.2	617.2	614.4	2.8	600.9	600.6	0.3	
Implicit price deflator for GNP (1968=100).....	109.6	110.4	110.0	0.4	111.3	110.8	0.5	112.8	111.2	1.6	113.0	111.7	1.3	111.6	110.9	0.7	
Civilian labor force (millions of persons).....	74.5	74.9	75.0	-0.1	75.3	75.5	-0.2	75.5	75.8	-0.3	75.8	76.1	-0.3	75.4	75.5	-0.1	
Employment (millions of persons).....	70.7	71.4	71.3	0.1	72.0	71.9	0.1	72.3	72.4	-0.1	72.5	72.9	-0.4	72.1	72.2	-0.1	
Unemployment (millions of persons).....	3.8	3.5	3.6	-0.1	3.3	3.6	-0.3	3.2	3.4	-0.2	3.3	3.2	0.1	3.3	3.3	0.0	
Unemployment rate (percent).....	5.1	4.6	4.8	-0.2	4.3	4.7	-0.4	4.2	4.4	-0.2	4.4	4.2	0.2	4.4	4.6	-0.2	

*Exogenous variables.

Sources: U.S. Department of Commerce, Office of Business Economics and U.S. Department of Labor, Bureau of Labor Statistics.

fitted, is a more adequate test of the model's performance than are the forecasts made for 1953-64. Moreover, 1965 presented something of a challenge to econometric models because special account had to be taken of a number of unusual events.

A dock strike occurred early in the first quarter reducing the volume of imports and exports. At the same time, production of autos picked up sharply following the auto strikes in late 1964. Steel producers and users continued to accumulate inventories in

anticipation of a steel strike. During the third quarter of the year, excise taxes on a number of consumer goods—mainly consumer durables—were removed, lowering prices paid by consumers. Apart from the further reduction in income tax rates in 1965, personal

Table 5.—Predicted and Actual Values for Endogenous Variables, 1965¹

	1964-- 4th Qtr. Actual	1st Quarter			2d Quarter			3d Quarter			4th Quarter			Year		
		Predicted	Actual	Difference	Predicted	Actual	Difference	Predicted	Actual	Difference	Predicted	Actual	Difference	Predicted	Actual	Difference
Gross national product components, billions of 1958 dollars:																
Personal consumption expenditures																
Automobiles and parts	24.8	29.6	30.2	-0.6	28.0	29.1	-1.1	29.4	30.7	-1.3	30.5	30.4	0.1	29.4	30.1	-0.7
Durable goods other than automobiles and parts	33.1	34.4	34.3	.1	34.8	34.3	.5	36.3	35.7	.6	36.0	37.5	-1.5	35.9	34.4	.5
Nondurable goods	171.8	174.8	173.2	1.6	175.6	176.4	-.6	179.2	177.8	1.4	181.0	181.0	.0	177.8	177.1	.5
Services, excluding housing	90.5	91.9	90.8	1.1	92.2	91.9	.3	94.4	92.9	1.5	94.3	93.6	.7	93.0	92.3	.7
Fixed investment, nonresidential	59.6	61.0	62.5	-1.5	63.2	63.7	-.5	63.6	66.0	-2.4	65.5	67.8	-2.3	63.4	65.0	-1.6
Residential structures, nonfarm	29.0	23.8	23.8	.0	23.6	23.9	-.3	23.4	23.3	.1	22.3	22.6	-.3	23.2	23.3	-.1
Change in business inventories	7.1	7.7	8.6	-.9	6.4	6.3	.1	6.5	7.3	-.8	5.5	6.5	-1.0	6.8	7.9	-1.1
Imports of goods and services	6.8	5.3	5.0	.3	5.3	5.8	-.5	5.3	5.0	.3	5.4	6.0	-.6	5.3	5.4	-.1
Imports of other goods and services	23.2	21.2	22.8	-.6	24.7	26.1	-1.4	25.0	27.1	-2.1	25.0	26.9	-.9	24.5	25.7	-1.2
Gross private output, excluding housing services	478.4	488.8	490.4	-1.6	494.1	494.7	-.6	500.8	502.3	-1.5	504.9	512.0	-7.2	497.1	498.6	-2.7
Gross private output at capacity	552.0	558.7	556.0	2.7	566.4	567.0	-.6	569.4	571.3	-1.9	577.2	578.3	-1.1	567.7	569.2	-1.5
Implicit price deflators (1958=100):																
Personal consumption expenditures	107.7	108.1	108.0	.1	108.9	108.7	.2	109.1	109.0	.1	109.6	109.3	.3	108.9	108.8	.1
Durable goods other than automobiles and parts	100.0	100.3	100.0	.3	100.8	99.7	1.1	99.4	98.0	1.4	100.0	97.1	2.9	100.1	98.7	1.4
Nondurable goods	103.3	103.9	103.6	.3	103.7	103.5	.2	107.1	107.2	-.1	107.9	107.7	.2	106.9	106.7	.2
Services, excluding housing	113.8	117.1	117.0	.1	118.6	119.7	-.1	119.8	119.9	-.1	120.7	120.6	.1	119.0	119.2	-.2
Fixed investment, nonresidential	105.6	107.3	107.0	.3	108.6	107.3	1.3	109.6	107.4	2.2	110.0	108.0	2.0	109.1	107.4	1.7
Residential structures, nonfarm	113.6	112.6	114.2	-1.6	113.5	115.1	-1.6	114.3	117.1	-2.8	113.7	117.0	-3.3	114.1	116.1	-2.0
Gross private output, excluding housing services	107.7	108.5	108.0	.5	109.4	108.9	.5	109.9	109.2	.7	110.8	109.4	1.4	109.6	108.9	.7
Income and related items, billions of dollars:																
Nonwage personal income	123.3	124.7	124.4	.3	126.8	128.5	-1.7	128.4	130.7	-2.3	128.9	130.0	-1.1	127.2	129.2	-2.0
Wage and salary disbursements and other labor income	359.7	368.2	364.4	3.8	374.5	371.7	2.8	383.1	377.4	5.7	385.2	387.0	-1.8	380.2	375.5	4.6
Profits tax liability	28.1	28.5	28.5	.0	29.2	29.8	-.6	29.8	30.1	-.3	29.8	31.1	-1.3	29.3	30.1	-.8
Undistributed profits and inventory valuation adjustment	19.1	22.9	24.2	-1.3	22.5	23.5	-1.0	22.1	24.2	-2.1	23.5	24.2	-.7	22.1	24.1	-2.0
State unemployment insurance benefits	2.4	2.3	2.4	-.1	2.0	2.2	-.2	2.0	2.2	-.2	2.0	2.0	.0	2.0	2.2	-.2
Wage rate, hours worked and output per man-hour, private sector:																
Annual wage rate, thousands of dollars	5.625	5.683	5.643	.040	5.771	5.727	.044	5.861	5.705	.156	5.867	5.781	.086	5.820	5.728	.092
Index of weekly hours worked, 1957-59=100	.985	.989	.988	.001	.988	.988	.000	.989	.989	.000	.989	.990	-.001	.989	.990	-.001
Index of output (excluding housing services) per man-hour, 1957-59=100	1.200	1.219	1.222	-.003	1.224	1.227	-.003	1.232	1.244	-.012	1.235	1.251	-.016	1.220	1.230	-.010
Monetary variables:																
Interest rate, 4-6 month commercial paper, percent	4.06	4.47	4.30	.17	4.47	4.38	.09	4.40	4.38	.02	4.46	4.47	-.01	4.42	4.38	.04
Yield, corporate bonds (Moody's) percent	4.38	4.66	4.55	.11	4.73	4.58	.15	4.79	4.66	.13	4.86	4.77	.09	4.76	4.69	.07
Mortgage yield, secondary market, FHA-insured new homes, percent	5.45	5.47	5.45	.02	5.50	5.45	.05	5.53	5.45	.08	5.57	5.49	.08	5.52	5.46	.06
Liquid assets of households, billions of dollars	320.8	322.7	322.9	-.2	324.6	323.9	.7	325.7	328.5	-2.8	335.0	339.0	-4.0	341.8	344.9	-3.1
Miscellaneous:																
Net stock of fixed investment, nonresidential, billions of 1958 dollars	468.1	472.8	472.7	.1	478.0	478.0	.0	483.0	489.7	-6.7	488.4	489.4	-1.0	480.8	481.0	-.2
Durable manufacturers' new orders per quarter, billions of 1957-59 dollars	58.1	61.6	62.1	-.5	63.2	62.1	1.1	64.5	63.5	1.0	64.3	65.4	-1.1	63.4	63.3	.1
Durable manufacturers' shipments per quarter, billions of 1957-59 dollars	58.6	58.9	60.1	-1.2	58.8	59.0	-.2	60.4	61.6	-1.2	62.7	62.4	.3	59.7	61.0	-1.3
Durable manufacturers' unfilled orders, end of quarter, billions of 1957-59 dollars	51.4	56.7	53.8	2.9	60.5	58.5	2.0	64.1	57.2	6.9	68.0	60.2	7.8	62.3	58.7	3.6
Private nonfarm housing starts, thousands of units	1,593.0	1,501.5	1,450.0	51.5	1,485.0	1,524.0	-39.0	1,422.0	1,431.0	-9.0	1,415.0	1,537.7	-122.7	1,467.0	1,495.7	-28.7

Note.—All data not specifically noted are at seasonally adjusted annual rates.

1. Items shown in table 4 are not repeated here.

SOURCE: U.S. Department of Commerce, Office of Business Economics and Bureau of the Census; U.S. Department of Labor, Bureau of Labor Statistics; Board of Governors, Federal Reserve System.

tax payments dropped from the somewhat inflated levels of the first half, which were associated with the under-withholding of taxes in 1964.

Fortunately, an econometric model is sufficiently flexible to make allowances for special factors of the kind just described. In an actual forecasting situation, such factors must, of course, be anticipated and quantified along with the usual exogenous variables. In testing the model over a past period, as with the OBE model, the task is made easier by the existence of *ex-post* information regarding the special factors. But most special elements cannot be isolated with precision even in retrospect. For example, in the present instance available data do not clearly indicate how much inventory buildup was due to the anticipation of a strike and how much was "normal." It is usually possible, however, to prepare at least a crude estimate of the special factors.¹⁰

It may be noted in this connection that the 1953-64 forecasts discussed previously were not adjusted for special factors other than through the use of the "dummy" variables appearing in the auto and inventory equations and through allowance for changes in taxes.

Tables 4 and 5 present in full detail the outputs of the model by quarter and for the year as a whole, together with corresponding actual values and errors of prediction. Table 4 presents GNP and its components in current dollars, income and reconciliation items, and certain supplementary items including labor force and employment data. Table 5 gives endogenous variables not shown in table 4.

10. Specifically, the following adjustments were made: To allow for abnormal auto purchases in the first quarter, a "dummy" variable—which is included in the auto equation to take care of strike situations—was assigned a value of one, adding \$1.9 billion more to consumer purchases than the equation would otherwise have yielded. Similarly, \$2.0 billion was temporarily added to inventories to allow for unusual steel and auto inventory buildup. An estimated reduction in imports during the first quarter and a subsequent makeup in the second, associated with the dock strike, were similarly incorporated.

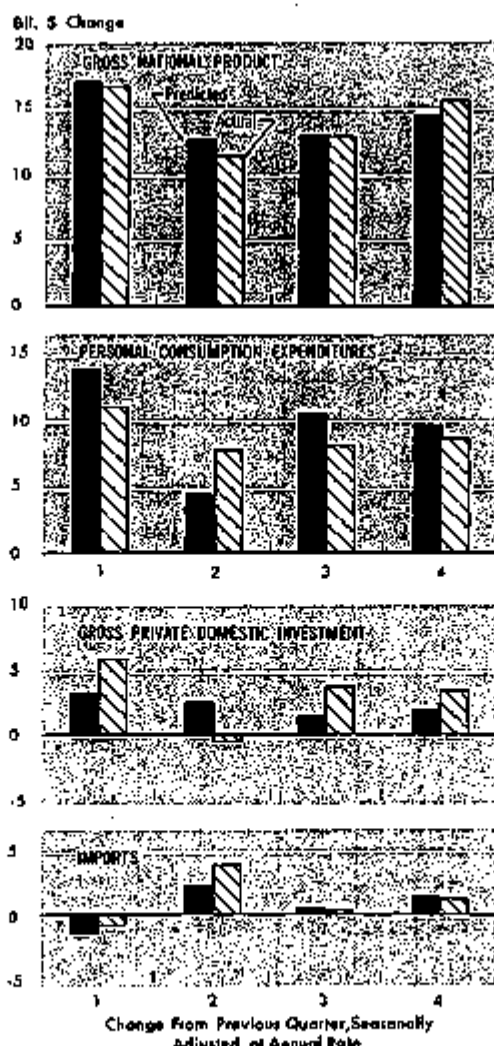
Amounts of \$2.5 billion, \$3.0 billion, and \$0.5 billion were added to the personal tax function for the first, second, and third quarters respectively. The implicit price deflators for "other" durables and total private output were reduced after the second quarter by 1.7 and 0.3 points respectively, on the assumption that the reductions in excise taxes were fully passed on to consumers; indirect business taxes were reduced by \$1.6 billion.

1965 performance

The model closely depicted the degree and pattern of economic expansion during the year. It yielded a GNP of \$677.4 billion for the year as a whole, or \$1.1 billion above the actual level. This represents an error of 2.3 percent in predicting the change in GNP from 1964, the actual change being \$47.6 billion. The error in predicting the change from the fourth quarter 1964 to the fourth quarter 1965 was only \$0.5 billion. As shown in chart 10, the model results gave a good depiction of the general pattern of quarterly GNP changes over the course of the year. This pattern was characterized by

CHART 10

Predicted Versus Actual Quarterly Changes in GNP and Major Components, 1965



U.S. Department of Commerce, Office of Business Economics

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large changes for the first and final quarters and somewhat more moderate gains for the intervening periods. The major components did not do quite as well on either an annual or a quarterly basis.

Table 4 shows that GNP was slightly overestimated for each quarter of the year, as has been the tendency since the Korean war. This reflects mainly a pattern of overestimating personal consumption expenditures. Not all consumption components were overestimated, auto purchases being the notable exception. Errors in individual investment components, though usually negative, were relatively small, except for the underestimate of inventory change in the fourth quarter, when actual inventories rose by an exceptional \$10.1 billion.

Personal income was overestimated, particularly in the third and fourth quarters. Positive errors centered in wage income and are attributable to an increasingly overestimated wage rate. Positive errors in wages were partly offset by underestimates of nonwage personal income. Predicted corporate profits (including inventory valuation adjustment), which in the model are inversely related to the wage rate, were also below actual levels.

GNP in 1958 dollars, unlike current-dollar GNP, was slightly underestimated for the first three quarters of the year and substantially so—by \$7.2 billion—for the fourth quarter. This reflects excessive price increases predicted by the model. The implicit GNP price deflator determined by the model was consistently higher than the actual, and markedly so by the fourth quarter. As shown in forecasts for earlier years, prices have been difficult to predict, though not always for the same reasons. In the present case, excessive price gains yielded by the model are clearly associated with overestimation of the wage rate.

Because the price results were not very satisfactory, another forecast was made with actual price deflators replacing those predicted by the equations. Since in this version prices were assumed to be exogenous, the model was reduced in scope to predicting real quantities on the product side.

Table 6 shows the main results for this alternative forecast. Interestingly, the behavior of current-dollar GNP and its major components was little affected during the first three quarters by making prices exogenous. However, GNP in 1958 dollars was estimated above the actual level in each quarter in this version. In the fourth quarter, current-dollar GNP was \$2.6 billion higher than before; that is, additional real output more than offset the reduction in price level.

In both versions of the forecast, the unemployment rate approximated the sharp decline that took place over the year. In the full version, the rate did not fall quite to the actual fourth quarter level, while in the exogenous price version it dropped below. The

lower unemployment rate in the exogenous price version reflects a larger gain in employment—virtually the same as the actual increase—associated with the greater rise in real output. In both cases, expansion of the labor force was somewhat underestimated.

It seems fair to say that this particular forecast has been improved by making prices exogenous. Whether this would be generally the case in actual *ex-ante* forecasting depends, of course, on how well independent price projections can be made.

Further research

The inadequacies of wage and price determination in the model point up the need to improve the specifications of the wage and price functions. This is a major challenge facing all econometric

model builders. Apart from this, further work is required in several areas to improve the OBE model. These areas include the monetary equations, the equations for inventories, man-hours, and imports. In addition, a number of the present equations show evidence of nonrandom residuals, suggesting the need for improved specifications.

Beyond this, the usefulness of the OBE model would be increased by further adapting it for policy purposes. This, as has been noted, entails the introduction of more policy variables and also the provision of endogenous explanations for as many nonpolicy variables as possible. In this connection, the major task ahead is the development of an endogenous function for fixed investment.

Table 6.—Predicted and Actual Major Forecast Items, 1965: Exogenous and Endogenous Price Versions

(Billions of dollars seasonally adjusted at annual rates, unless not applicable)

	1964-4th Qtr.	1Q			2Q			3Q			4Q			Year		
		Prices		Actual	Prices		Actual	Prices		Actual	Prices		Actual	Prices		Actual
		Exog-enous	Endog-enous		Exog-enous	Endog-enous		Exog-enous	Endog-enous		Exog-enous	Endog-enous		Exog-enous	Endog-enous	
Gross national product.....	641.1	659.0	659.1	657.6	670.9	670.5	663.8	683.6	683.3	681.8	700.3	697.7	697.2	678.4	677.4	676.3
Personal consumption expenditures.....	406.0	419.9	419.8	416.9	424.1	424.1	424.5	436.0	434.4	432.5	446.1	444.9	441.6	431.3	430.8	428.7
Residential structures.....	25.7	27.2	25.8	27.7	27.7	27.4	28.0	28.0	27.5	27.7	26.8	26.4	27.3	27.4	27.0	27.6
Fixed investment, nonresidential.....	63.5	65.3	65.6	66.9	68.1	68.6	68.4	68.8	69.7	70.9	71.3	72.5	73.0	69.2	69.1	69.8
Change in business inventories.....	7.6	8.8	8.3	8.8	7.4	6.9	4.4	7.6	7.0	7.6	8.5	7.0	10.1	8.1	7.3	8.2
Net exports of goods and services.....	8.9	4.4	6.6	6.0	10.1	10.1	8.0	9.6	9.4	7.4	8.7	8.8	8.9	8.5	8.7	7.1
Disposable personal income.....	446.4	463.5	463.9	451.4	469.2	468.9	466.6	475.0	474.4	471.2	487.4	484.4	480.3	468.8	467.6	465.3
Gross national product (1958 dollars).....	584.7	599.1	598.3	597.7	603.4	603.0	603.6	615.0	611.2	613.0	626.6	617.2	626.4	611.5	608.9	606.6
Implicit price deflator for GNP (1958=100).....	100.6	110.0	110.4	110.0	110.8	111.2	110.8	111.2	111.8	111.2	111.7	112.0	111.7	110.9	111.6	110.9
Civilian labor force, millions of persons.....	74.5	74.9	74.9	75.0	75.3	75.2	75.5	75.8	75.8	75.8	76.9	76.8	76.1	75.4	75.4	75.6
Employment, millions of persons.....	70.7	71.6	71.4	71.3	72.1	72.0	71.9	72.4	72.3	72.4	72.9	72.5	72.9	72.2	72.1	72.2
Unemployment, millions of persons.....	3.8	3.4	3.5	3.6	3.2	3.3	3.6	3.1	3.5	3.4	3.0	3.3	3.2	3.2	3.3	3.4
Unemployment rate, percent.....	5.1	4.6	4.6	4.8	4.3	4.3	4.7	4.2	4.2	4.4	4.0	4.4	4.2	4.3	4.4	4.6

SOURCE: U.S. Department of Commerce, Office of Business Economics and U.S. Department of Labor, Bureau of Labor Statistics.

(Appendix A follows)

APPENDIX A

ESTIMATED EQUATIONS

Numbers in parentheses under coefficients are standard errors of the coefficients. Key to variables and other abbreviations follows equations.

I. GNP Component Equations

(1) Personal consumption expenditures, automobiles and parts

$$C_a = -134.0 - 11.0 \frac{P_c}{P_c} + .104 \frac{Y-T}{P_c} + 129.0(h_u)_{-1} + 1.85d_u; \\ (.14) \quad (.64) \quad (.006) \quad (.19.4) \quad (.30)$$

$$TSLS, \bar{R}^2 = .91, \bar{S} = 1.0, D.W. = 1.25.$$

(2) Personal consumption expenditures, durables other than automobiles and parts

$$C_d = 28.0 + .060 \frac{Y-T}{P_d} - 65.2 \frac{P}{W} + .060 \left(\frac{L_h}{P_d} \right)_{-1}^{ess}; \\ (.07) \quad (.002) \quad (.15.2) \quad (.008)$$

$$TSLS, \bar{R}^2 = .98, \bar{S} = .5, D.W. = .88.$$

(3) Personal consumption expenditures, nondurables

$$C_n = 31.1 + .252 \frac{Y-T}{P_n} + .210 \frac{1}{8} \sum_{i=1}^8 (C_n)_i; \\ (.15) \quad (.025) \quad (.083)$$

$$TSLS, \bar{R}^2 = .995, \bar{S} = 1.0, D.W. = 1.23.$$

(4) Personal consumption expenditures, services (except housing)

$$C_s = -44.2 + .069 \frac{Y-T}{P_s} + .476 \frac{1}{8} \sum_{i=1}^8 (C_s)_i + .347N; \\ (.06) \quad (.015) \quad (.161) \quad (.118)$$

$$TSLS, \bar{R}^2 = .998, \bar{S} = .5, D.W. = 1.13.$$

(5) One-family housing starts, private nonfarm ¹¹

$$HS_1 = -768 + .832 (HS_1)_{-1} - .113 (HS_1)_{-2}; \\ (7.9) \quad (.137) \quad (.081)$$

$$- 43.9 (r_m)_{-1} + 1530 \left(\frac{R_h}{q_h} \right)_{-1} - .0216 V_{-2}^{ess}; \\ (13.9) \quad (486) \quad (.0235)$$

$$OLS, \bar{R}^2 = .92, \bar{S} = 54.5, D.W. = 1.96.$$

11. Prior to 1961 the average Treasury bill yield, lagged two, three, and four quarters, is used in place of $(r_m)_{-1}$ with coefficient of -.848.

(6) Residential structures, nonfarm

$$I_h = -.14 + .001022 \left[.41 \left(\frac{c_h}{q_h} HS \right) + .49 \left(\frac{c_h}{q_h} HS \right)_{-1} + .10 \left(\frac{c_h}{q_h} HS \right)_{-2} \right] - \frac{.19d_1}{(.21)} + \frac{.04d_2}{(.23)} + \frac{.38d_3}{(.24)} + I_{h-1};$$

$$OLS, \bar{R}^2 = .93, \bar{S} = .4, D.W. = 1.36.$$

(7) Fixed investment, nonresidential

$$I_p = 11.0 + .804I_p + .108(\Delta X)_{-1} + .524(I_p^* - I_p)_{-1} + .163t + .14C_a;$$

(1.5) (.045) (.026) (.126) (.012)

$$OLS, \bar{R}^2 = .96, \bar{S} = 1.0, D.W. = 1.25.$$

(8) Change in business inventories

$$I_i = 49.9 + .232(X - I_i - C_i) + .363(I_i)_{-1} - .354 \sum_{j=0}^{i-1} (I_i)_j + .215(U_i)_{-1} + .72t + 4.34d_i;$$

(0.2) (.044) (.084) (.053) (.064) (.18) (.86)

$$TSLS, \bar{R}^2 = .81, \bar{S} = 1.6, D.W. = 2.04.$$

(9) Imports other than crude materials and foodstuffs

$$F_{it} = 15.5 + .0573 \frac{F-T}{P_t} - \frac{49.4}{(17.0)} \frac{P}{W};$$

(0.1) (.0026)

$$TSLS, \bar{R}^2 = .97, \bar{S} = .6, D.W. = .60.$$

(10) Imports of crude materials and foodstuffs

$$F_{cm} = 3.94 + .0027 \left(\frac{pX}{P_t} \right)_{-1};$$

(.03) (.0005)

$$OLS, \bar{R}^2 = .35, \bar{S} = .2, D.W. = 1.18.$$

II. Price and Wage Rate Equations

(11) Implicit price deflator, gross private output, except housing services

$$p = .226 + 1.305 \frac{1}{3} \sum_{i=0}^{i-2} \left(\frac{W - W_i}{X} \right)_i + .00208 \left(\frac{X}{\bar{X}_e} \right)^{0.2} \sum_{j=0}^{i-1} \Delta(X - I_i)_j + .00113t;$$

(.0003) (.076)

$$TSLS, \bar{R}^2 = .996, \bar{S} = .004, D.W. = .99.$$

- (12) Implicit price deflator, personal consumption expenditures, durables other than automobiles and parts

$$p_{ad} = -.095 + .77\Delta p + 1.08(p_{ad})_{-1} + .000154(U_d)_{-1};$$

(.0004) (.18) (.04) (.000045)

$$TSLS, \bar{R}^2 = .96, \bar{S} = .003, D.W. = 1.39.$$

- (13) Implicit price deflator, personal consumption expenditures, nondurables

$$p_n = -.019 + .95\Delta p + 1.016(p_n)_{-1};$$

(.0005) (.18) (.012)

$$TSLS, \bar{R}^2 = .99, \bar{S} = .003, D.W. = 1.86.$$

- (14) Implicit price deflator, personal consumption expenditures, services (except housing)

$$p_s = -.118 + .155w + 1.56 \frac{C_s}{C_T};$$

(.002) (.004) (.41)

$$TSLS, \bar{R}^2 = .99, \bar{S} = .010, D.W. = .28.$$

- (15) Implicit price deflator, residential structures, nonfarm

$$q_h = .491 + .115w;$$

(.002) (.003)

$$TSLS, \bar{R}^2 = .97, \bar{S} = .011, D.W. = .73.$$

- (16) Implicit price deflator, fixed investment, nonresidential

$$q_p = .023 + 1.39\Delta p + .976(q_p)_{-1};$$

(.0006) (.23) (.008)

$$TSLS, \bar{R}^2 = .997, \bar{S} = .004, D.W. = 1.83.$$

- (17) Wage rate (private sector)

$$\frac{w - w_{-1}}{w_{-1}} = -.015 + .0106 \left[\sum_{i=0}^3 \left(\frac{N_L - N_w - N_s}{N_L} \right)_i \right]^{-1}$$

$$+ .877 \sum_{i=0}^3 \left(\frac{p_c - p_{c-1}}{p_{c-1}} \right)_i + .00128(P_c - P_{c-1}) - .311 \frac{w_{-1} - w_{-2}}{w_{-1}};$$

(.293) (.00047) (.119)

$$TSLS, \bar{R}^2 = .57, \bar{S} = .012, D.W. = .77.$$

III. Employment, Weekly Hours, and Labor Force Equations

- (18) Average weekly hours (employees)

$$h_w = .821 + .223 \frac{X}{X_c} - .00041t;$$

(.0005) (.018) (.00004)

$$TSLS, \bar{R}^2 = .92, \bar{S} = .003, D.W. = 1.71.$$

(19) Man-hours per unit of capacity output

$$\frac{h(N_w - N_e + N_s)}{X_t} = \frac{.1684}{(.0002)} - \frac{.00109t}{(.00008)} + \left[\frac{.125}{(.013)} - \frac{.00148t}{(.00059)} \right] \frac{X^* - X_t}{X_t} \\ + \frac{.0579}{(.0121)} \frac{X - X^*}{X_t}; \quad X^* = \frac{1}{6} (3X_{-1} + 2X_{-2} + X_{-3}); \\ TSLS, \bar{R}^2 = .99, \bar{S} = .00105, D.W. = .51.$$

(20) Civilian labor force

$$\frac{N_L}{N'} = \frac{.5753}{(.0030)} + \frac{.183}{(.126)} \frac{N_w + N_s}{N'} + \frac{.00047t}{(.00023)} + .83(\hat{w}_{N_L})_{-1}; \\ TSLS(TN), \bar{R}^2 = .13, \bar{S} = .0035, D.W. = 1.85.$$

IV. Nonwage Income Components Equations

(21) Nonwage personal income

$$P = \frac{55.6}{(.2)} + \frac{.149P_e}{(.062)} + \frac{.794t}{(.035)} + DIV; \\ TSLS, \bar{R}^2 = .98, \bar{S} = 1.6, D.W. = .66.$$

(22) Corporate profits and inventory valuation adjustment

$$P_e = \frac{215.0}{(.2)} + \frac{.275}{(.053)} \left[\frac{CGP}{pX} \right] pX - \frac{19.4}{(11.4)} \frac{w}{p} - \frac{550}{(140)} \frac{h(N_w - N_e + N_s)}{X} - \frac{40.0}{(7.8)} \frac{X_e}{X} - D_{ac}; \\ OLS, \bar{R}^2 = .99, \bar{S} = 1.5, D.W. = .59.$$

(23) Dividends

$$DIV = \frac{-.576}{(.038)} + \frac{.0418P_e}{(.0102)} + \frac{.897DIV_{-1}}{(.033)}; \\ TSLS, \bar{R}^2 = .99, \bar{S} = 2, D.W. = 2.72.$$

V. Monetary Equations

(24) Interest rate (short-term), 4-6 month commercial paper

$$r_s = \frac{1.06}{(.03)} - \frac{.214R_{-1}}{(.102)} + \frac{.977r_s}{(.087)}; \\ OLS, \bar{R}^2 = .92, \bar{S} = .24, D.W. = .80.$$

(25) Yield, corporate bonds (Moody's)

$$r_2 = \frac{.243}{(.017)} + \frac{.082r_s}{(.030)} + \frac{.885(r_2)_{-1}}{(.039)}; \\ TSLS, \bar{R}^2 = .96, \bar{S} = .11, D.W. = 1.45.$$

(26) Mortgage yield, FHA-insured new homes

$$r_m = -.591 + .198r_L + .739(r_m)_{-1};$$

(.015) (.070) (.077)

$$TSLS, \bar{R}^2 = .96, \bar{S} = .10, D.W. = .97.$$

(27) Liquid assets of households

$$L_h = -154 + 1.084p_c(C' + C_t) + 152 \frac{1}{r_L} + .85(\hat{u}_{L_h})_{-1};$$

(2.4) (.047) (50)

$$TSLS(TN), \bar{R}^2 = .93, \bar{S} = 2.4, D.W. = 1.98.$$

VI. Miscellaneous Equations

(28) Capital consumption allowances, constant dollars (fixed nonresidential capital stock)

$$D_p = -4.89 + .0340(K_p)_{-1};$$

(.04) (.0010)

$$OLS, \bar{R}^2 = .96, \bar{S} = 2, D.W. = .15.$$

(29) Gross private output at capacity¹²

$$X_s = 3.734(10)^{.00222t} [(K_p)_{-1}]^{.306} [.97(N_L - N_s)]^{.595}$$

(30) Personal tax and nontax payments¹³

$$T_p = a_0 + a_1 Y$$

(31) Corporate profits tax liability¹³

$$T_c = b_0 + b_1 P_c$$

(32) Indirect business tax and nontax liability

$$T_i = -9.39 + .125_t(X - I_t) + .112t;$$

(.10) (.011) (.050)

$$TSLS, \bar{R}^2 = .995, \bar{S} = .7, D.W. = .36.$$

(33) State unemployment insurance benefits

$$TR_u = -1.60 + 1.11(N_L - N_u - N_s);$$

(.05) (.06)

$$TSLS, \bar{R}^2 = .88, \bar{S} = .3, D.W. = .66.$$

12. For explanation of how equation is estimated, see Appendix B.
 13. Coefficients are determined on basis of recent tax behavior and modified when required to comply with changes in the tax laws.

(34) New orders, manufacturers' durables

$$O_d = -1.10 + .955 \frac{P_c}{P_{md}}; \\ (.33) \quad (.052)$$

$$TSLS, \bar{R}^2 = .87, \bar{S} = 2.3, D.W. = 1.12.$$

(35) Shipments, manufacturers' durables

$$S_d = 13.8 + .917\Delta(O_d)_{-1} - .202\Delta\left(\frac{U_d}{S_d} O_d\right)_{-1} + .715(O_d)_{-2}; \\ (.3) \quad (.142) \quad (.076) \quad (.048)$$

$$OLS, \bar{R}^2 = .84, \bar{S} = 1.8, D.W. = 1.33.$$

(36) Unfilled orders, manufacturers' durables

$$U_d = -.38 + .92(O_d - S_d) + (U_d)_{-1}; \\ (.11) \quad (.04)$$

$$OLS, \bar{R}^2 = .93, \bar{S} = .8, D.W. = 1.32.$$

VII. Identities

$$(37) \quad h = \frac{h_w(N_w - N_p) + h_p N_p}{N_w - N_p + N_p}$$

$$(38) \quad HS = HS_p + HS_n$$

$$(39) \quad p_a C_a + p_{ad} C_{ad} + p_n C_n + p_s C_s + p_r C_r + q_n I_n \\ + I_{nr} + q_r I_r + p I_i + e_i - p_i (F_{ir} + F_{im}) + F_s + G = GNP$$

$$(40) \quad GNP = pX + W_e + p_r C_r$$

$$(41) \quad W + P + P_c - DIV - i_c - i_v + T_b + TR_b + D_{ac} \\ + D_{na} + T_i - S_e + SD = GNP; |SD| \leq 4.0, |SD - SD_{-1}| \leq 1.0$$

$$(42) \quad Y = W + P$$

$$(43) \quad T = T_p - TR_n - TR_e + T_i$$

$$(44) \quad w = \frac{W - W_e}{h_w(N_w - N_p)}$$

$$(45) \quad p_c = \frac{p_n C_n + p_{ad} C_{ad} + p_n C_n + p_s C_s + p_r C_r}{C' + C_r}$$

$$(46) \quad C' = C_a + C_{ad} + C_n + C_r$$

$$(47) \quad S_p = Y - T - p_c(C' + C_r) - i_c - TR_r$$

$$(48) \quad S_e = P_c - T_e - DIV$$

$$(49) \quad K_p = (K_p)_{-1} + .25I_p - D_p$$

Key to Abbreviations

(All variables except interest rates are seasonally adjusted. All components of the national income and product accounts are at annual rates; other flow variables are at quarterly rates unless otherwise noted. Variables preceded by * are exogenous.)

C'	Personal consumption expenditures, except housing services, billions of 1958 dollars.
C_a	Personal consumption expenditures, automobiles and parts, billions of 1958 dollars.
$*C_h$	Average cost per new private nonfarm housing unit started, in thousands of dollars.
CGP	Corporate gross product, billions of dollars (ratio CGP/pX is assumed exogenous).
C_n	Personal consumption expenditures, nondurables, billions of 1958 dollars.
C_{nd}	Personal consumption expenditures, durables other than automobiles and parts, billions of 1958 dollars.
$*C_h$	Personal consumption expenditures, housing, billions of 1958 dollars.
C_s	Personal consumption expenditures, services (except housing), billions of 1958 dollars.
d_1, d_2, d_3	Seasonal dummy variables, housing expenditures equation; $d=1$ in quarter corresponding to subscript, 0 otherwise.
$*d_s$	Dummy variable for auto equation (-1 during strike quarter; +1 following strike quarter; +1 in 1955 to reflect abrupt credit and taste changes; 0 otherwise).
$*d_i$	Dummy variable for inventory equation (-1 during strike quarter; +1 before and after strike; 0 otherwise).
$*D_{ac}$	Capital consumption allowances, corporate sector.
$*D_{anc}$	Capital consumption allowances, noncorporate sector.
dev	Deviation from least squares linear trend.
DIV	Dividends, billions of dollars.
D_p	Capital consumption allowances, constant dollars, fixed nonresidential capital stock, quarterly rate, billions of 1958 dollars.
e_t	Discrepancy in jumpoff quarter between change in business inventories in current dollars and pI_t .
$*F_c$	Exports, billions of dollars.
F_{ct}	Imports other than crude materials and foodstuffs, billions of 1958 dollars.
F_{im}	Imports of crude materials and foodstuffs, billions of 1958 dollars.
$*G$	Government purchases of goods and services, billions of dollars.
GNP	Gross national product, billions of dollars.
h	Average weekly hours index, private sector (1957-59=1.000).
$*h_n$	Average weekly hours index, self-employed (1957-59=1.000).
h_w	Average weekly hours index, private employees (1957-59=1.000).
HS	Private nonfarm housing starts, in thousands at annual rate.
$*HS_m$	Number of new 2 or more family units started, in thousands at annual rate.
HS_s	Number of new single-family units started, in thousands at annual rate.
$*i_c$	Interest paid by consumers, billions of dollars.
$*i_g$	Net interest paid by government, billions of dollars.
I_p	Residential structures, nonfarm, billions of 1958 dollars.
$*I_{pf}$	Residential structures, farm, billions of dollars.
$*I_{pr}$	Residential construction expenditures on other than new units (additions and alterations, etc.), billions of 1958 dollars.
I_t	Change in business inventories, billions of 1958 dollars.
I_p	Fixed investment, nonresidential, billions of 1958 dollars.
I_p^0	Actual plant and equipment outlays in billions of dollars deflated by q_{p-2} .
$*I_p^1$	Anticipated plant and equipment outlays; first anticipations in billions of dollars deflated by q_{p-2} .
K_t	End of quarter net stock of plant and equipment, billions of 1958 dollars.
L_h	End of quarter liquid assets held by households (currency+demand and bank savings deposits+savings and loan shares), in billions of dollars.
$*N$	Total population in millions.
$*N'$	Population, ages 18-64 in millions.
$*N_s$	Self-employed, millions.
$*N_g$	Civilian government employment, millions.
N_L	Civilian labor force, millions.
N_w	Civilian wage and salary employment, millions.

O_d	Durable manufacturers' new orders, billions of dollars deflated by p_{ms} .
p	Implicit price deflator, gross private output, except housing services (1958=1.000).
P	Nonwage personal income (sum of proprietors' income, rental income of persons, dividends, and personal interest income), billions of dollars.
$*p_c$	Implicit price deflator, personal consumption expenditures, automobiles and parts (1958=1.000).
p_c	Implicit price deflator, personal consumption expenditures (1958=1.000).
$*p_i$	Implicit price deflator, imports (1958=1.000).
p_n	Implicit price deflator, personal consumption expenditures, nondurables (1958=1.000).
p_{nd}	Implicit price deflator, personal consumption expenditures, durables other than automobiles and parts (1958=1.000).
$*p_h$	Implicit price deflator, personal consumption expenditures, housing (1958=1.000).
p_s	Implicit price deflator, personal consumption expenditures, services (except housing) (1958=1.000).
$*p_{wd}$	Wholesale price index, durable goods (1957-59=1.000).
P_i	Corporate profits and inventory valuation adjustment, billions of dollars.
q_h	Implicit price deflator, residential structures, nonfarm (1958=1.000).
q_n	Implicit price deflator, fixed investment, nonresidential (1958=1.000).
$*R$	End of quarter excess reserves as percent of total reserves.
$*r_d$	Federal Reserve average discount rate (percent).
R_h	BLS consumer rent index (1957-59=1.000).
r_L	Percent yield, corporate bonds (Moody's).
r_m	Percent yield, secondary market, FHA-insured new homes.
r_s	Rate, 4-6 month commercial paper (percent).
S_c	Undistributed profits and inventory valuation adjustment, billions of dollars.
S_d	Manufacturers' shipments, durable goods, billions of dollars deflated by p_{ms} .
SD	Statistical discrepancy, billions of dollars.
$*S_g$	Subsidies less current surplus of government enterprises, billions of dollars.
S_p	Personal saving, billions of dollars.
t	Time in quarters (1953-I=1.0).
$*T_s$	Employer contributions for social insurance, billions of dollars.
T_p	Profits tax liability, billions of dollars.
$*T_c$	Personal contributions for social insurance, billions of dollars.
T_i	Indirect business tax and nontax liability, billions of dollars.
T_n	Personal tax and nontax payments, billions of dollars.
$*TR_s$	Business transfer payments, billions of dollars.
$*TR_f$	Personal transfer payments to foreigners, billions of dollars.
$*TR_o$	Transfer payments to persons, except State unemployment insurance benefits, billions of dollars.
TR_u	State unemployment insurance benefits, billions of dollars.
U_d	Unfilled manufacturers' orders, durable goods at end of quarter, billions of dollars, deflated by p_{ms} .
$(\hat{a}_{L_n})_{-1}$	Estimate of lagged disturbance, liquid assets equation.
$(\hat{a}_{W_n})_{-1}$	Estimate of lagged disturbance, labor force equation.
V	Number of vacant nonfarm housing units, end of quarter, in thousands.
w	Annual wage rate, private sector, in thousands of dollars.
W	Wage and salary disbursements and other labor income, billions of dollars.
$*W_g$	Government compensation, billions of dollars.
X	Gross private output, except housing services, billions of 1958 dollars.
X^*	Planned private output, billions of 1958 dollars.
X_c	Gross private output at capacity, billions of 1958 dollars.
$Y-T$	Disposable personal income, billions of dollars.
OLS	Ordinary least-squares estimate.
$TSLs$	Two-stage least-squares estimate.
TN	Equation is estimated using Theil-Nagar transformation of variables. ¹⁴
$D.W.$	Durbin-Watson statistic: Test for serial correlation of residuals.
\bar{R}^2	Adjusted coefficient of determination.
\bar{S}	Adjusted standard error of estimate.

14. See H. Theil and A. L. Nagar, "Testing the Independence of Regression Disturbances," *Journal of the American Statistical Association*, LVI (December 1961), pp. 793-806.

APPENDIX B

Method Used To Obtain Estimates of Parameters

With a few exceptions, the two-stage least-squares (TSLS) method was used to obtain estimates of the parameters in the equations of the model. Among the various methods available to obtain consistent estimates in an interdependent simultaneous system, this is by far the simplest to apply and has been shown in past studies to yield acceptable results.

The application of TSLS to obtain parameter estimates for the present model differs in one important respect from the more usual way the method is used. The customary procedure has been to obtain the first-stage computed values by regressing all "right-side" endogenous variables on all predetermined variables in the system or on some selected subset of them. In the present instance, because of the large number of predetermined variables relative to the number of observations, the computed values were obtained by regressing the endogenous variables on leading principal components of the predetermined set.¹⁵ In brief, principal components are certain linear combinations of the variables in the predetermined set which capture in condensed form the essential information contained in the full array. The method was adopted primarily for convenience and to avoid computational difficulties associated with large systems that have strong correlations among the predetermined variables. It was found that 10 principal components were sufficient and all but exhausted the information contained in the full set. Thus, all first-stage regressions were based on the set of derived variables, 10 in number.

Specific mention should be made of the method used to obtain parameter

estimates for capacity output given by equation (29). The parameter estimates were obtained indirectly as follows:

(1) A linear homogeneous Cobb-Douglas production function was first fitted, using actual private output, man-hours employed, and utilized capital approximated by multiplying the Wharton School Index of capacity utilization by total capital. The computed elasticities were used in the next step.

(2) The difference $\log X_c - a \log K - (1-a) \log [0.7 (N_z - N_d)]$ was then regressed on time, where $X_c = \frac{X}{WSI}$.

(3) The constant term and the coefficient for time from (2) and the

elasticities from (1) were incorporated into equation (29). Values computed from this equation for X_c , rather than X_c , are used to define capacity output. The usual supplementary statistics are not shown for this equation.

The above procedure was adopted to remove the unrealistic fluctuations in capacity output obtained by direct application of the Wharton School Index. The fluctuations arise primarily because of the inapplicability, in a strict sense, of the index, which is based on indexes of industrial production, to a GNP concept of total private output. Although this procedure removes the fluctuations, it does not correct for possible bias in the estimate of capacity level.

APPENDIX C

Treatment of the Statistical Discrepancy in the Model

In designing econometric models, it has been customary to include equations to account for all but one income item—usually corporate profits—which is then determined residually through the constraint, found in the national income and product accounts, that income plus reconciliation items equal product. This implies that values for the usual reconciliation items, including the statistical discrepancy, must be introduced. In general, the discrepancy is set at the previous period's or some other predetermined level.

This procedure has the serious drawback that the residual item must bear the brunt of errors made elsewhere in the model—errors that are by no means necessarily offsetting. To alleviate this difficulty, the present model uses a different approach. Behavioral equations initially determine all income elements, including corporate profits. Then, in order to avoid overdetermina-

tion implied by adding the income-product identity (there being then more equations than unknowns), the statistical discrepancy is defined as a new variable, rather than assuming a pre-assigned value. This makes the discrepancy, rather than an income component, the residual.

Clearly, the discrepancy cannot be allowed to vary without limit since the essential identity between income and product must hold. Thus, two constraints are introduced to limit the behavior of the discrepancy: (1) Its level is constrained to vary within the approximate historically observed range, from -\$4.0 billion to +\$4.0 billion, and (2) its maximum allowable quarterly change is set at \$1.0 billion. Movements of the discrepancy beyond either of these limits give rise to an excess that must be eliminated.

The method of elimination is as follows: Adjustments are made on cer-

15. See, for example, T. W. Anderson, *An Introduction to Multivariate Statistical Analysis* (John Wiley and Sons, 1958), pp. 272-291, and T. Klock and L. B. M. Monies, "Simultaneous Equations Estimation Based on Principal Components of Predetermined Variables," *Econometrica*, January 1960, pp. 45-61.

tain income-determining equations, namely those for the wage rate, man-hours, corporate profits, and personal nonwage income, by shifting the constant terms in these equations by amounts sufficient to eliminate the excess discrepancy when the model is again solved.

These adjustments have two effects. First, they serve to reconcile income and product by spreading the excess residual element among the major income components rather than concentrating it in profits. Second, because of feedbacks to spending primarily via disposable income, the levels of income and product are adjusted to the point where they are consistent with both the necessary income-product identity (within the above-stated discrepancy limit) and the requirements of the model.

The relative amounts of adjustment introduced into the four equations are somewhat arbitrary; they were determined so that the resulting income increment or decrement would be distributed among wages, corporate profits, and personal nonwage income in amounts based partly on the relative sizes of these components, and partly on the residual character of corporate profits and entrepreneurial income. Equal weight was attached to the wage rate and to man-hours in affecting wage payments.

The decision to confine adjustments exclusively to income items was not arbitrary. It was based on examination of the multipliers implied by the given system of equations. This examination showed that errors made on the product side of the accounts tend to bring about similar errors on the income side in both magnitude and direction. Errors on the income side, however, do not affect product commensurately, and they therefore tend to affect the statistical discrepancy. Thus, the procedure adopted is based mainly on what is expedient to bring about a desired change in the statistical discrepancy. However, apart from expediency, it appears quite likely that the income equations are more precarious, which also suggests the adoption of the adjustment procedure, at least on a provisional basis.

APPENDIX D

Method Used to Solve the Model¹⁶

Examination of the model equations shows that many of them are nonlinear in endogenous variables. Given such a system, the usual methods of matrix inversion for the solution of a set of linear equations cannot be applied, and an alternative method must be employed.

The particular method used to solve the model was originally suggested by Professor Klein and consists, essentially, in the separation of the equations of the system into two parts. The first step in the solution is to introduce provisional values for a select set of variables in one of the parts—values from the previous period are satisfactory for this purpose. The variables selected are such as to remove the nonlinearities in the set so that a linear solution method can be applied to obtain provisional values for the remaining unknowns.

These values are then introduced into the second part, which can then also be solved as a linear set. The solution yields revised values for the variables initially introduced into the first set. After a number of iterations (usually five to seven), the process converges to a solution for the entire set of equations.

Solution is assumed complete when two successive iterations yield values of the endogenous variables that differ by no more than 0.05 percent. This ensures that all variables are computed correctly to the degree of precision given by the corresponding data. Because of the treatment accorded to the statistical discrepancy (see Appendix C), the iteration procedure is carried out twice for each quarter in which the discrepancy does not fall within the prescribed limits.

APPENDIX E

Constant and Time Trend Adjustments

In most of the equations of the model there is evidence of serial correlation of residuals. This indicates either imperfections in specification or autocorrelated errors of measurement in the data. To minimize forecasting errors resulting from serial correlation, it is reasonable to adjust the intercept or constant term of each equation showing such correlation so that the computed value of the dependent variable coincides with the last observed value or with the average of recently observed values.

In simulating with the model, wherever serial correlation was believed to be present, constants were adjusted as just described. Either the value of the last quarter (prior to forecast) or average values of the last four quarters were used depending on whether serial

correlation was deemed to be strong or moderate. For equations estimated with the Theil-Nagar transformation, this procedure was superseded by appropriate explicit treatment of the lagged residual.

In applying the model beyond the sample period, it is also appropriate to examine residuals in equations containing trend terms to see if there has been a shift in the trend. Thus, in making the 1965 forecast, a trend correction was applied to the man-hours equation because of an apparent drift of the residuals over the recent period prior to 1965.

16. Solutions were carried out on an IBM 704 computer by means of a program called OMNITAB. Cf. J. Eisenrath, G. C. Ziegler, G. G. Messina, P. J. Walsh, and R. J. Harbold, *OMNITAB: A Computer Program for Statistical and Numerical Analysis*, National Bureau of Standards Handbook 101 (U.S. Government Printing Office, 1966).